POSTSTRUCTURALISM AND DECONSTRUCTION:
A MATHEMATICAL HISTORY

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ABSTRACT: Explaining his love of philosophy, Slavoj Žižek notes that he 'secretly thinks reality exists so that we can speculate about it'. This article takes the view that links between mathematics and continental philosophy are part of reality, the reality of philosophy and its history, and hence require speculation. Examples from the work of Jacques Derrida and Henri Poincaré are discussed.

KEYWORDS: Poststructuralism; Derrida; Philosophy of Mathematics

Examples are not lacking of philosophers whose outlook was inspired or in some way influenced by thinking about the meaning of mathematics: Wittgenstein, Husserl, Russell, before them Peirce, and more recently Badiou, to name only the clearest cases in point. Philosopher and mathematical logician Jean Cavaillès was undoubtedly influential on post-war philosophy in France, and the effects of his mathematical critique of the philosophy of the subject, especially of Kant and Husserl, carried an important impulse from formalist mathematics to twentieth-century French philosophy. Even Heidegger, although critical of formal logic and technical reason (like some mathematicians of his day), followed the debate on the nature of mathematical knowledge and may have been provoked by it — especially by the controversy regarding the time-continuum — to attempt in his *Being and Time* (1927) to

"destroy" traditional metaphysics and thus transgress the philosophical options that found themselves at loggerheads over the question of the foundations of mathematics.

The link works in the other direction, too, although it has become something of a rarity to find mathematical articles that contain references to Kant, Fichte, Schopenhauer and Nietzsche. During the foundational debate in the 1920s, one could see this in the articles of Hermann Weyl and L.E.J. Brouwer; Weyl was actively interested in phenomenology and maintained correspondence with Husserl, while Kurt Gödel is known to have been a serious reader of Kant and Husserl. Much earlier, Hermann Grassmann (a major influence on Alfred North Whitehead) had built on the ideas of his father, Justus Grassmann, who wrote under the influence of Schelling.[1] These links have not been severed, even if they remain unstated. Thus, for example, the philosophically reticent Bourbaki collective was, according one of its members, a "brainchild of German philosophy". One finds the influence of Husserl and Heidegger in the mathematical essays of Gian-Carlo Rota, and at least implicitly in the work of Petr Vopěnka on alternative set theory. It may not be the norm, but examples of cross-fertilization are not as difficult to find as the oversimplified binarism of "two cultures" would have us believe.

My goal here is to indicate the relevance of mathematics to several important points made by Jacques Derrida. A number of Derrida's arguments bear resemblance to critiques of logic and excesses of formalist mathematics. These objections hark back to the ideas of "intuitionist" mathematicians who — some, I think, under the influence of German romantic idealism — rebelled in the early 1900s against the hegemony of formal logic and the symbolic reduction of all thought to computation. The situation is not quite that simple, since Derrida apparently also employs certain ideas of formalist mathematics in his critique of idealist metaphysics: for example, he is on record saying that "the effective progress of mathematical notation goes along with the deconstruction of metaphysics."[2]

Derrida's position can, I think, be interpreted as a sublation of two completely opposed schools in mathematical philosophy. For this reason it is not possible to reduce it to a readily available philosophy of mathematics. One could perhaps say that Derrida continues and critically reworks Heidegger's attempt to "deconstruct" traditional metaphysics, and that his method is more "mathematical" than

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[1] I am grateful to Professor Arran Gare for pointing out the influence of Hermann Grassmann on A. N. Whitehead, and the role of Schelling's philosophy in the work of Hermann's father, Justus Grassmann. See: Marie-Louise Heuser, "Geometrical Product – Exponentiation – Evolution. Justus Günther Grassmann and Dynamist *Naturphilosophie*" in: G. Schubring (ed.), *Hermann Günther Graßmann (1809-1877): Visionary Mathematician, Scientist and Neohumanist Scholar* (Dordrecht: Kluwer, 1996) pp. 47-58.

[2] Jacques Derrida, *Positions* (Chicago, Chicago University Press, 1981), p. 35.

Heidegger's because he has at his disposal the entire pseudo-mathematical tradition of structuralist thought. He has implied in an interview given to Julia Kristeva that mathematics could be used to challenge "logocentric theology," and hence it does not seem unreasonable to try looking for mathematical analogies in his philosophy.

A word of caution, though. The similarities I will outline here are similarities of argumentative *techniques*, not of philosophical outlooks. The analogies — which are informed and limited by my own interpretive ability and my belief that mathematics and continental philosophy are deeply related — are not to be confused with the gross misstatement that "mathematicians have done it all." Finally, no a posteriori observed or reconstructed homology of argumentation can imply a philosophical affinity between thinkers as different as are, say, Derrida and Poincaré. Nevertheless, I believe that looking for such common traits as we can find in the history of ideas may be helpful in bridging the enormous gap that seems to separate mainstream science and contemporary continental philosophy.

Let us start with the problem of identity. It is perhaps a commonplace of "postmodern" thought to say that "identity is not present to itself," that it is in some sense secondary to the concept of difference. Derrida is frequently credited with bringing up this point, and we will see in moment how his argument proceeds. For now we can say that according to this view logic itself rests on ungrounded assumptions about absolute self-identity of objects, assumptions which cannot be formally deduced in any way and thus sit as an unseemly blemish on the face of formal methodology.

To the best of my knowledge, an argument of this type was first made in Ficthe's *Basis for the Entire Theory of Science* (1794), where he wrote that "if the proposition $A=A$ is certain then the proposition 'I am' must also be certain",[3] thus implying that the principle of identity is not *logically* certain but inevitably involves a *hypothetical* judgment on the part of the subject. These ideas were further elaborated by Schelling — for instance, in his critique of Hegel — but it was probably Nietzsche who summed it up in the manner most relevant to mathematics: "logic (like geometry and arithmetic) only holds good for assumed existences which we have created."[4]

So the problem has been around for a while. However, to most scientists, arguments that bring into question formulas like $A=A$ are bound to seem silly. How can anyone in their right mind claim that this tautological principle is not logically grounded? Is this type of sophism worth the attention of a "serious" scientist?

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[3] Johann Gottlieb Fichte, *Sämthliche Werke*, Vol. 1 (Berlin, Veit, 1845-6), p. 95.
[4] Johann Gottlieb Fichte, *Sämthliche Werke*, Vol. 1 (Berlin, Veit, 1845-6), p. 95.
Oscar Levy (ed.), *The Complete Works of Friedrich Nietzsche*, Vol. 15 (New York, Macmillan, 1909-13), p. 33.

Apparently it is; or was. The same point has been made by Poincaré, a scientist as serious as they get. In an article published in 1891, he analyzes the concept of identity as it is explained in Euclid's *Elements*. (The analysis applies to any formal axiomatic system, but Poincaré presumably chose the *Elements* because it was at that time the most accessible example of a formalized theory.) According to Euclid, figures are to be regarded as equal if one can be superimposed over the other. Poincaré asks: What does Euclid mean by "superimposed"? In order to superimpose one object over another, one would have to be displaced until they coincide. Poincaré continues: "But how must it be displaced? If we asked this question, no doubt we should be told that it ought to be done without deforming it, as an invariable solid is displaced."[5]

Thus "objects" are implicitly assumed to be invariable bodies; solid bodies. The axioms of geometry already contain an irreducible assumption which does not follow from the axioms themselves. Axiomatic systems provide us with "faulty definitions" of objects, definitions that are grounded not in formal logic but in a hypothesis — a "prejudice" as Hans-Georg Gadamer might say — that is *prior* to logic. As a corollary, our logic of identity cannot be said to be necessary and universally valid. "Such axioms," says Poincaré, "would be utterly meaningless to a being living in a world in which there are only fluids."[6]

Since, if we accept this, identity cannot be deduced from the formal rules of the structure under consideration, the attribution of identity always already involves an agency capable of hypothesizing it. This, I believe, is similar to the point made by Fichte in the statement quoted above. It also seems likely to be the reason why Schelling asserted that the principle of identity cannot be deduced and that the agency able to "provide" it must be taken as the irreducible first principle of philosophy. To be sure, this leads to other difficulties, the notion of "absolute ego" and "self-consciousness in general," which take us somewhat far from the realm of mathematics. It is perhaps better to stay with Poincaré.

Poincaré's answer to this problem is to invoke a specific sort of intuition, a "prereflective" familiarity with our life world due as much to the body as it is to the mind. "To geometrize," says Poincaré, "is to study the properties of our instruments, that is, of solid bodies."[7] The intuition of continuity exists in our minds prior to experience, but it is our involvement with the world that provides us with the

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[5] See Henri Poincaré, "Les géometries non-euclidiennes," *Rev. Génerale Sci. Pures Appl.* **2** (1891), 769-74. English translation quoted from Arthur Miller, *Imagery in Scientific Thought* (Cambridge, Mass., MIT Press, 1986), p. 19.

[6] *Ibid.*

[7] From Poincaré's 1912 lecture at the University of London. Quoted from Miller, *Imagery in Scientific Thought*, p. 24.

opportunity to "choose" among the many possible groups of continuous transformations the one that best fits our concrete bodily experiences. This is not quite an intuition in Kant's sense, although Poincaré did in a sense defend the Kantian view that mathematical inferences are "synthetic," that unlike logical inferences they involve a "creative" act on the part of the subject.[8] But Poincaré's intuition is not transcendental or ahistorical. Nor is it like Husserl's phenomenological intuition, since for Poincaré intuition is fundamentally informed by the bodily aspect of our experience, by our own *embodiment* as it were. (There is, however, a certain similarity here with Husserl's notion of the "living body moving trough the life world.")

This intuitive familiarity with the world is in a sense impersonal or trans-personal — it is, as Poincaré insists, the conquest of the human race rather than of the individual — but it is at the same time the basis for individual interpretive contributions since, for Poincaré, identity always occurs as a *continually motivated* hypothesis by the subject. Poincaré's intuition seems to be a "Bergosonian" awareness of the unanalyzable continuity of our movements; and its necessity is motivated, supported, not by invoking the sciences of perception, psychology or physiology, but rather in a "phenomenological" manner. (Poincaré is of course not a phenomenologist, but his ideas, his emphasis on the bodily aspect our understanding of spatiality, seem to reflect an attitude similar to the one found in the essays of Jan Patočka.)[9]

Being supported by "phenomenological" arguments, this intuition ultimately tells us nothing about space itself, but only about our common understanding of it; so that for Poincaré, surprisingly, space could have as many dimensions as there are muscles in the human body. Finally, such an intuitive understanding may evolve along with our bodies/minds and our familiarity and practical involvement with the life-world. In this sense one could perhaps say that it is related to what Heidegger would later call an intuition "derived" from the structure of existential understanding. Of course, just like Poincaré is not a phenomenologist, Heidegger is not a mathematician, and there are enormous differences in their goals, depth of their argumentation, etc. However, there does seem to be something vaguely mathematical about this structure

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[8] Poincaré's attacks on *logistique* were on occasion very entertaining. Logical inferences are so "sterile," "mechanical," that "if we prefer, we might imagine a machine where we should put axioms at one end and take theorems at the other, like that legendary machine in Chicago where pigs go in alive and come out transformed into hams sausages." See Henri Poincaré, *Science and Method* (New York, Dover, 1952), p. 147.

[9] Jan Patočka, *Body, Community, Language, World* (Chicago, Open Court, 1998). I am grateful to Professor Ivan Chvatik of the Center for Theoretical Study, Prague, for introducing this book to me.

of existential understanding, even for Heidegger. Although this "mathematical" structure cannot be fully formalized, Heidegger writes that "the mathematical project is the anticipation of the essence of things, of bodies; thus the basic blueprint of the structure of every being and its relation to every other being is sketched out in advance"; he notes that the mathematical "first opens up the domain in which things show themselves," and that "in this projection is posited that which things are taken as."[10]

Unlike Poincaré, who died approximately at the time when Heidegger published one of his earliest articles, "New investigations in logic" (1912), Derrida is aware of Heidegger's argument from *Being and Time* that intuition is already derived from the structure of existential understanding, and he does not allow for intuition as a fundamental ground. Let us take Poincaré's analysis and see where it might lead if we prohibit any mention of a grounding intuition. In this case, we are left with the following: the concept of identity is construed negatively, as invariance under transformations; hence, identity can be established only a posteriori, after an object has been given a chance to change; therefore, identity is, as a concept, subordinate to the concept of difference, through which it subsists; and, finally, identity can never be established absolutely, it is not a logically grounded concept.

This looks a little like Derrida's "deconstruction of identity," although he typically speaks of "iterability" instead of "invariance." That is a function of his interest in the philosophy of language as opposed to geometry, but iterability can be understood as *semantic* invariance of a speech-act under any number of repetitions. (Whether or not he had any interest in geometry per se, it must be said Derrida's first major work, and one of the least well known, is a long introduction to Husserl's *Origins of Geometry*. Besides, Poincaré also used the term "iterability," precisely in the context of his discussion of identity.)

Consider an example where Derrida applies this kind of argument. It will help us understand the historical connections to which he may implicitly be pointing. In a polemic with John Searle, Derrida attacks speech-act theory precisely on the grounds that it takes the concept of iterability for granted, just as axioms of geometry make an ungrounded assumption about the invariance of objects. In speech-act theory, the meaning of an utterance is basically determined by reference to a set of prescribed conventions valid in a particular linguistic community. These conventions, understood as rules that cover all possible circumstances and even speakers' possible intentions, provide the analog of a set of axioms from which the meaning of each utterance can be deduced. It is, in other words, something like "Euclid's *Elements of*

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[10] Martin Heidegger, *Basic Writings* (New York, Harper & Row, 1977), p. 268.

Speech," an axiomatic system of conventions that supposedly govern all competent linguistic acts.

Derrida accepts all of this as the basis for discussion with Searle. But he attacks Searle in the way in which Poincaré attacked formalizations of geometry. Identity of geometric objects, according to Poincaré, is not granted *by the axioms*: it is *assumed* from the start, being contained in our notion of "object" as a thing invariant under certain types of transformations. Replacing invariance under displacement by semantic invariance, that is, iterability, Poincaré's argument can be applied to Searle. Indeed, Derrida undermines the core of speech-act theory by challenging the assumption that utterances are necessarily indefinitely iterable given the set of conventions ("rules," "axioms") specified by speech-act theory. His elaborate critique appears in *Limited, Inc.*, where Derrida plays many interesting tricks on Searle (maybe too many). An entertaining part of this exchange is that, in his reply to Derrida, Searle reaches precisely for Frege — one of Poincaré's original targets — to defend his views:

> Without this feature of iterability there could not be the possibility of producing an infinite number of sentences with a finite list of elements; and this, as philosophers since Frege have recognized, is one of the crucial features of any language. [...] Any conventional act involves the notion of the repetition of the same.[11]

This is exactly what Derrida is citing as the ungrounded assumption. Searle's reply is therefore, ironically, simply a "repetition of the same," without any additional argument, of the very point at which Derrida attacked his theory. It seems that Searle did not know about Poincaré's polemic against logical reductions of mathematics (Frege included). This is unfortunate because it would have been quite interesting to see how he could possibly prove that speech-act theory solves the problem of the identity of mathematical objects. If it is the case, as some philosophers have claimed, that "Fregeanism" has ruled the analytic curriculum on the American campuses, it is unsurprising that Derrida's question about the invariability of speech-acts must have seemed just as bizarre to Searle as Poincaré's critique seemed to formalists and logicists at the turn of the twentieth century. Indeed, Searle soon broke off the debate. It is not clear whether Derrida was aware of the history of the arguments he used, but the similarity, the structural homomorphism with the debate between Poincaré and the logicians, is at least a curious coincidence. And it is certainly not the only coincidence one may notice. Let us look a bit further.

As we saw in his discussion of identity, Poincaré argued that the axioms of geometry (although, as noted before, his objections apply to any axiomatic system)

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[11] John Searle, "Reiterating the Differences: A Reply to Derrida," *Glyph* **1** (1977), pp. 199, 207.

implicitly rely on assumptions about the nature of objects which they are supposed to "define," thus giving us a vicious circle that defines nothing. Poincaré was more generally concerned with various cases of such a *petitio principii*. Another objection he made to axiomatic systems is directly pertinent to what would now be regarded as the poststructuralist critique of structuralism.

In part inspired by the work of linguist Ferdinand de Saussure, structuralist ideas grew into a movement that had considerable impact in disciplines far from linguistics. Simplifying in the extreme — and ignoring some important questions as to whether Saussure himself intended structural linguistics to be simplified in this manner — we could say that the basic idea is as follows. The units ("signifiers") of a structure have no identity outside of the formal structure under consideration. Their identity is derived from structural relationships with other units within the structure. Furthermore, the relationship between the signifier (the formal structural unit) and the signified (the physical or mental representative of the structural unit) is arbitrary.

I am deliberately formulating it in this manner in order to emphasize the connection with the formal methodology of mathematics. It is certainly the case that mathematicians are not interested in the "nature" of the objects under consideration, but in their structural relationships. This was repeatedly emphasized by Poincaré, Weyl, and most famously by the founder of the formalist school of mathematics, David Hilbert. A well known (though possibly apocryphal) statement describes the philosophical attitude of Hilbert's *Foundations of Geometry* (1899) as always being ready to replace "points, lines, circles" by "tables, chairs, beer-mugs." Objects themselves are not of interest — only their structural relationships within a formal structure to which they belong. Using this analogy, one can transform critiques of such a "structuralism" in mathematics into critiques of structuralism in general.

On a first glance, the "formalist" approach seems to avoid the problems pointed out by Poincaré. Identity of objects, in this setup, becomes in some sense irrelevant: we programmatically make no assumptions about them. Rather, we are hoping that the structure itself will provide a formal surrogate of the objects' identities. This opens up other difficulties, because such a definition of identity is once again circular: it involves a *petitio principii* since it defines an object by invoking the totality to which the object belongs. Poincaré called such definitions *impredicative*. They are especially problematic when they occur in structures that are in some sense "generative," that is, structures that, like language and formalized arithmetic, are capable of generating new units according to some rule. If a new unit is introduced into the structure, I must in principle reset the process of signification to "derive" the identity of all the units.

What is there to guarantee that this process will preserve the identity of the "old" units?

Nothing in the structure itself necessarily guarantees it. Simplifying in the extreme for the sake of an example, one could say that the value of a monetary unit depends not on the physical representative of it, but on its relationship with the other elements in the economic structure. Clearly, unpleasant experiences with impredicatively defined structural units of the monetary system are quite possible, which is why monetary units are under the control of a central authority. The problems are not *necessary*, but the possibility that identity becomes "fluid" due to the introduction of new units cannot be excluded.

Saussure himself (who seems in part to have been inspired by political economy) was definitely aware of this problem. Although the statement was apparently suppressed by the editors of his posthumously published *A Course on General Linguistics* (1916), in a later critical edition one can find the following statement, which could just as easily have been made by Poincaré:

> What has escaped philosophers and logicians is that from the moment a system of symbols is independent of the objects designated it is itself subject to undergoing displacements that incalculable for the logician. [12]

Poincaré used the so-called Berry's paradox to illustrate this possibility. I can name a number by saying that it is the smallest positive integer not nameable in less than thirty syllables. If there is a totality of positive integers "out there," then among those that are not nameable in less than thirty syllables, there is the smallest one. So I named a definite number, and I used less than thirty syllables to do it. I named an integer in the way that by its very definition it cannot be named. This is Berry's paradox. One way of resolving it would be to say that my assertion about the paradoxical integer prompted a resignification across the structure, so that my very utterance, the very *thought* of it actually, changes the meaning assigned to each unit. Most people find this absurd. I could also say that the introduction of this utterance into our linguistic structure changes the meaning assigned to the concept of nameability. Many people would find this just as bizarre. For Poincaré, nothing in the formal structure itself can ensure the desired "immutability." (Indeed, if mathematical meaning were "immutable," mathematicians would not bother with the unusual activity of finding new proofs of old theorems: a new proof is always a recontextualization of the "old" result, it is not a mechanical repetition.)

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[12] See Ferdinand de Saussure, *Cours de linguistique génerale*, Édition Critique, ed. by Rudolf Engler, Vol. 2 (Wiesbaden, Harrasowitz, 1967-74), p. 23. English translation quoted from Manfred Frank, *What is Neostructuralism?* (Minneapolis, University of Minnesota Press, 1989), p. 426.

There is, however, a way out: it is indeed the standard resolution of the problem. I must carefully distinguish "metalanguage," in which one can make assertions about the language of integers, from the "object language," that is, the language of integers itself. In this manner I effectively introduced a higher principle that prohibits the use of sentences which mix the two levels of language and produce unpleasant paradoxes. But note that to do this I had to introduce a *higher* principle, equivalent to the "central bank" in the monetary analogy, something on top of what my original structure provides on its own. In the absence of this extra assumption, things could fluctuate in an apparently unpredictable manner. The necessity of introducing such a principle to guarantee the "conservation" of identity is explicitly stated by Jean Piaget, in his book *Structuralism*:

> These properties of conservation along with the stability of boundaries despite the construction of indefinitely many new elements presuppose that structures are self- regulating.[13]

This is where Derrida begins his attack on structuralism. He envisions a kind of "universal" generative structure, which he calls text-in-general. It is the structure of all structures, so to speak. There is nothing beyond it, or, more precisely, nothing that we can *know*. That, at any rate, is how I understand Derrida's celebrated but perplexing assertion that "there is nothing outside of the text": nowledge involves *justification*, and therefore language, writing, *text*.

This ultimate structure is capable of producing indefinitely many new units: new theorems are proved, new books are written, new combinations of the letters of the genetic code appear upon the act of making love, new materials are produced in the grammar of chemical reactions, and so on. *What* gives us the right to assume, as Piaget does, that the impredicatively defined units of this structure have a stable identity? What is this principle of self-regulation, which Derrida famously calls "the center," and how can we say that we *know* anything about it?

From this question, one can hop to the poststructuralist territory quite easily. Where is this center? If it is outside of the "text," then we cannot say that we know anything about it: it resides beyond our power of justification and therefore outside our knowledge. If we want to say that we can know about it, then it must be within the "text in general." But in that case, being part of the text, it is itself an impredicatively defined unit of the text, and hence cannot itself be guaranteed to have any stable identity. It cannot be guarded from uncontrollable semantic transformations, and therefore cannot provide the stability that it is supposed to guarantee. In other words, by definition of the center, it cannot be the center. And so

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[13] Jean Piaget, *Structuralism* (New York, Harper & Row, 1970), p. 14.

our ultimate text appears to be "decentered," and all identity could well dissolve into a fog, an endless play of difference with no reassuring stability. Here is what Derrida says about it:

> The concept of a centered structure is in fact the concept of play based on a fundamental ground, a play constituted on the basis of a fundamental immobility and a reassuring certitude, which is itself beyond the reach of play.[14]

As we can see, this is a substantial radicalization of Poincaré's argument, but one that is in essence not very far from the "original." Derrida, unlike Poincaré, applies the same basic theorem to a much broader class of structures. For instance, his "deconstruction of the subject" follows a similar pattern. Simply said, my "I" — at least if understood as a kind of *cognition* of the self, as a reflection in the "text" — must also make a detour through the text in order to come into knowledge of itself. To know myself, to cognizantly ascertain my self- identity, I must be able to distinguish the "I" from an indefinite number of other units in the great text. But this is an impredicative definition: it invokes the totality of elements to which the "I" belongs. Therefore, stability of the identity derived in this manner cannot in principle guaranteed. It follows that the identity of the self can only be guaranteed by making a metaphysical assumption about the "center" which reassures me that my identity is in some sense grounded. (Fichte was certainly aware of this problem. For, to know myself, I would have to be able to distinguish myself from all that I am not: every determination is a negation. But I cannot do that since I am not in a position to know the indefinitely many things that could be not-I. This difficulty forced Fichte to abandon the notion that self-consciousness can be understood on the model of reflection. He argued that self-consciousness is grounded in something that is itself *not* reflected, something called "immediate self-consciousness," "non-positing consciousness," etc. This is of course metaphysical, but it does provide an alternative to Derrida's "deconstructed subject.")

With these considerations we are getting closer to understanding what Derrida might have meant when he wrote that mathematics could be used to challenge "logocentric theology." But in another statement, made during the aforementioned interview with Kristeva, he says specifically that the progress of mathematical *notation* goes along with a deconstruction of metaphysics. To clarify this, we must leave Poincaré behind and look at Hilbert's attack on idealism in mathematics.

In the first three decades of the twentieth century there was an important attempt to ground mathematics in the activity of the self-conscious subject. According to this view, whose most radical proponent was Dutch mathematician L.E.J. Brouwer, the

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[14] Jacques Derrida, *Writing and Difference* (Chicago, Chicago University Press, 1978), p. 279.

fundamental basis of mathematical activity is the construction of time-continuum in consciousness. This is related to Husserl's work, although it appears that Brouwer and Husserl came upon their ideas about the continuity of inner time independently and in different ways. Husserl's ideas about continuity of inner time, its construction as a "two-dimensional" retention/protention sequence, bears some resemblance to Weyl's interpretation of Brouwer's time-continuum as a collection of "double- sequences" of a special kind. It is up for debate how deep this connection is; it has been studied in detail by Mark van Atten.[15] At any rate, there are other potential points of contact between intuitionism and phenomenology; both are, for instance, concerned with effective and not purely formal demonstrability, the process of construction of objects in consciousness, and with the problem of what constitutes *evidence*.

But Brouwer was an extremely radical thinker who flirted with solipsism, for whom mathematics was an act of will of the mysterious "creative subject" that is "far from reasoning and words," a totally private languageless activity of the mind that has its basis in the construction of inner time. Logic, on the other hand, was for Brouwer a degenerate form of mathematics, a kind of ethnology that studies how people organize their thoughts: claiming that logic is the basis of mathematics is like claiming that "the human body is an application of the science of anatomy". Brouwer regarded any attempt at linguistic reduction of mathematics — especially of the continuity of inner time — as fundamentally impossible. Husserl, although very far indeed from such radicalism, certainly maintained an interest in the subject, in particular through his correspondence with Weyl, who for a while supported Brouwer's "revolution." (This "revolutionary" discourse led a certain Cambridge philosopher to brand Brouwer and Weyl as "Bolsheviks," to which Wittgenstein responded by calling the fellow a "bourgeois philosopher." So this debate was a serious one, complete with attempts at political discreditation.)

Hilbert was clearly on the side opposite to Brouwer and Weyl (who was Hilbert's student in Göttingen). He exclaimed that "Brouwer is not, as Weyl thinks, the revolution," and proceeded to gather around him a group of mathematicians and philosophers who would help establish what is now known as the formalist school of the philosophy of mathematics. Although it started by making references to a return to Kant — likely under the influence of the neo-Kantian philosopher Leonard Nelson who worked in Göttingen — the formalist school eventually abandoned

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[15] Mark van Atten, *Brouwer Meets Husserl. On the Phenomenology of Choice Sequences* (Berlin, Springer, 2006). See also M. van Atten, D. van Dalen, R. Tieszen, "Brouwer and Weyl: the phenomenology and mathematics of the intuitive continuum", *Philosophia Mathematica* **10**(3) (2002) 203-226.

"anthropocentrism" and concentrated on justification of "the method." This caused even Nelson to accuse Hilbert's school of "nihilism."

There was and still is a lot of confusion regarding just how "formalist" this school was, but its adversity to any discussion of the role of the subject in mathematics, as well as to phenomenological considerations and even to application of Friesean *Kritische Philosophie* in meta-mathematical discussions as advocated by Nelson, is beyond doubt. Logical consistency of formal systems became the most important issue, and clarifying the intuitive basis of mathematical activity, although originally an important presupposition of Hilbert's philosophy, took a back seat to technical questions.

There are certain reasons why this was a natural development, but we can leave that aside. The effects of this shift of emphasis are addressed in Husserl's *Crisis of European Sciences* in a section on "the loss of mathematical meaning through formalization and technization." There, he discreetly laments the prejudices of the positive sciences that have prevented Brouwer's and Weyl's notions from finding wider acceptance. Husserl avoids naming names, but Weyl was more outspoken: "If Hilbert's view prevails, then I see in this a decisive defeat for the philosophical attitude of pure phenomenology."[16]

What, then, was Hilbert's view? Despite frequently making references to Kant, Hilbert appears to have been of the opinion that the progress of mathematics must not be held back by philosophical debates as to the intuitive groundedness of formal theories. There is, however, something Kantian about his approach. Hilbert believed that part of mathematics, "real" mathematics, was intuitively given. This would be the ability to understand and manipulate finite strings of symbols, a kind of "semiotic intuition" that would replace the a priori intuitions of space and time. Just what this intuition is supposed to give is a question that is still debated. But the basic idea is not difficult to sketch out. Objects that are in a certain sense of "finitary" nature are intuitively graspable, "real." That includes, for instance, finite strings of symbols that one would write down in the course of proving a theorem. Formalized mathematics that deals with construts other than these "real" ones would then be something like Kant's ideal judgments — judgments that do not form a basis for determining any object as such, but are merely tools for the "orderly management of our understanding." That we interpret them *as* objects is a sort of "transcendental

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[16] Hermann Weyl, "Diskussionbemerkungen zu dem zweiten Hilbertschen Vortag über die Grundlagen der Mathematik," *Abhaundlungen aus dem Mathematischen Seminar der Hamburgischen Universität* **6** (1928), 86-88. English translation quoted from van Heijenoort, J., *From Frege to Gödel* (Cambridge, Mass., Harvard University Press, 1967), p. 484.

illusion," the effect of pure reason's necessary attempt to transcend what is given to it in experience.

The task that Hilbert's Program set for itself was to demonstrate, using only the methods of "real" mathematics — called metamathematics — that "ideal" mathematics, seen as a collection of finite strings of symbols (and therefore something to which "real" mathematics can legitimately be applied), does not contain mutually contradictory statements. Thus, metamathematics would be the global justifier of all local "formula games," that is, parts of ideal mathematics that proceed under the "pragmatic" attitude that as long as they are logically consistent they stand a chance of being useful to science. (One could perhaps look for the sources of Wittgenstein's concept of a language game in the vicinity of these ideas, especially after the viability Hilbert's Program was brought into serious doubt by Gödel.)

As an example of how this debate reverberated in France during the 1970s, let me quote Cornelius Castoriadis, who in *The Imaginary Institution of Society* (1975) writes that

> if mathematics were simply the ordered manipulation of signs […], statements and demonstrations would be only arrangements of different order iterations of the 'single' sign '.'; and the 'rules' determining what is a 'well-formed formula' and a 'demonstration' would, in fact, be simply the 'accepted' or 'chosen forms' of the spatial arrangement of points […]. We know that this is a chimera, pursued for some time by some great mathematicians, but abandoned for 40 years [i.e., since Gödel], and which has now reappeared as the broken-down horse mounted by successive waves—ethnological, linguistic, psychoanalytic, semiotic—of Parisian fashion. [17]

Going back to the foundational debate, one of the reasons that Husserl, Brouwer and Weyl objected to this approach is that Hilbert, rather than allowing room for a discussion of motivation of our interpretive practices with regard to these "ideal" objects — and mathematical *praxis* undoubtedly teems with heuristic interpretations of all kinds — required that all references to intuitive motivation of formal theories be suppressed since they are "so much anthropocentric garbage." Even though it does not seem to have been Hilbert's intention from the start, this requirement eventually led to the absurd accusation that mathematics was for Hilbert a collection of "meaningless formula games" justified only by their logical consistency. How this view came to be propagated is not of much importance for the present discussion. All we need to concentrate on is the following important point that Hilbert made in his critique of "anthropocentric" (intuitionist or idealist) views of mathematics:

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[17] Cornelius Castoriadis, *The Imaginary Institution of Society*, translated by Kathleen Blamey (Cambridge, Mass, MIT Press, 1987), p. 184 and 396n34.

> To make it a universal requirement that each individual formula be interpretable by itself [as Brouwer demanded] is by no means reasonable; on the contrary, a theory by its very nature is such that we do not need to fall back on intuition in the midst of an argument.[18]

Physicist Paul Dirac made a similar point in an article from 1931, and indeed put it into practice in the process of developing what is now known as Dirac's equation:

> The most powerful method of advance that can be suggested at present is to employ all the resources of pure mathematics in attempts to perfect and generalize the mathematical formalism that forms the existing basis of theoretical physics, and after each success in this direction, to try to interpret the new mathematical features in terms of physical entities.[19]

In this manner, intuition loses the primacy it has in idealist philosophy. Our "intuitive" understanding, even though part of it may be given prior to experience, appears to us also as a sense of familiarity transformed through *praxis*, and can sometimes be retroactively informed by theories constructed purely formally. Wittgenstein certainly argued that this is so: "mathematical proofs [...] lead us to revise what counts as the domain of the imaginable."[20] Heidegger was even more explicit about that and saw in it the reason for a decisive overcoming of traditional metaphysics:

> One of the burning questions concerns the limits and justification of mathematical formalism in contrast to the demand for an immediate return to intuitively given nature. [...] The question cannot be decided by way on an either/or, either formalism or immediate return to intuitive determination of things; for the nature and direction of the mathematical project participate in deciding their possible relation to the intuitively experienced and vice versa.[21]

With this in mind, we can begin to understand what Derrida means when he claims that *writing* should be regarded as a more appropriate subject of philosophical discussion than speech. Speech, according to him, and he argues such metaphors with a certain literary flourish, implies the presence of the speaker, of an *intended meaning*, a crystal-clear intention "present" prior to the act of linguistic articulation. In science, as

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[18] David Hilbert, "Die Grundlagen der Mathematik," *Abhaundlungen aus dem Mathematischen Seminar der Hamburgischen Universität* **6** (1928), 65-85. English translation quoted from van Heijenoort, *From Frege to Gödel*, p. 475.

[19] See Paul Dirac, "Quantized Singularities in the Electromagnetic Field," *Proc. Roy. Soc. London A* **133** (1931), 60-72.

[20] Ludwig Wittgenstein, *Philosophical Investigations* (Oxford, Blackwell, 1953), para. 517.

[21] Heidegger, *Basic Writings*, p. 270.

argued by Hilbert and Dirac the process of writing seems to be primary with regard to any such "presence." Scientific practice sometimes involves writing things down "in the dark"; what we mean by this piece of writing may become clear only in the process of writing or even after a "result" occurs. It does not mean that science is irrational and random, but perhaps some of its procedures could be described by Mike Leigh's directorial tenet: "to dicover what the film is about by making it."

On this view the objects of science, especially of mathematics, are not necessarily "present" and cannot be said to be *re*presented in mathematical writing; on the contrary, they sometimes emerge during the process of writing. In this manner, it seems that it is *writing*, rather than presence of an intention on the part of a mathematician, that is the crucial condition of possibility for the emergence of mathematical objects. In the language of literary theory, Brouwer's idealism was a case of the intentionalist fallacy: mathematical language was for him merely an imperfect vehicle for the transmission of intention ("transmission of the will", as he put it). Hilbert disagreed, and argued that mathematical language has a more fundamental role. Introduction of ideal elements is a legitimate procedure, even if it is unintentional and counter-intuitive or not justifiable by reference to an a priori intuition.

An easy example is the "imaginary unit," the square root of -1. The sixteenth century Italian algebraists who "discovered" it had no idea about such things. They simply solved some cubic equations and noticed that even though their formulas involved taking square roots of negative numbers, the solutions were formally correct. Today, of course, we would say that imaginary numbers are solutions of some equations, but in the sixteenth century such things were not regarded *as* objects. Cardano, Tartaglia, Bombelli and Ferrara had no idea about imaginary numbers to begin with, and could not interpret their mysterious formulas a *re*presentation of something, some idea, previously present in their minds. (Cardano spoke even of negative numbers as "fictional," *numeri ficti*.) It was only over the course of several centuries that these formal constructs were given the status of bona fide objects of scientific practice and they seem completely "normal" to us now. Simply said, one occasionally proceeds in a *purely formal* manner, without a clear intention, and at best only subsequently addresses the issue of meaning, of the presence of "physical" or "mental" referents of the theory.

Thus, a discussion of whether something is an intuitable object may turn out to be counterproductive, and at least in some cases hold the process of science hostage to historically contingent prejudices about intuitively apprehensible objects. Hilbert's school became even more radical and eventually rejected *all* hermeneutics of

mathematical writing, attempting to replace even the intuitive notion of truth by formal demonstrability. This attempt has not been a success, but it was nevertheless sufficiently influential to banish most forms of idealism from mathematics and replace it by linguistic considerations. It is perhaps this feature of formalism that most interests Derrida, who, like Hilbert (and Heidegger and Wittgenstein), wants to do away with idealist metaphysics.

Let us see what Derrida wants to do with this new idea, the notion that writing is in a sense the primary activity whose importance has been suppressed due to the symbolic value we grant speech (associating it with breath, life, presence). We already know that in the absence of a central principle, the meaning of textual units could float in a manner that cannot be systematically controlled. The possibility that meaning is affected each time a new piece of the text is written cannot be excluded in a "decentered" structure. This, of course, does not entail that my identity necessarily changes every time someone writes something down. But I cannot guarantee that I will *not* be affected by new pieces of the text-in-general, and Derrida correctly points out that philosophy cannot simply ignore this possibility. It is, in a sense, a political project: can we think decentered structures, that is, can we think structure-in-general without having to rely on the "central bank" that guarantees its stability?

Consider, then, the "worst case scenario," namely, that every piece of writing somehow affects the identity of the textual units. Is there any structure to the "liquid" semantics that results, or is it a totally unarticulated darkness in which all cats are gray? Derrida claims that this endless process of semantic transformation, which he calls *différance*, is not *astructural*; that it can be described, studied, thematized (even if it cannot be completely captured by any subject):

> the production of differences, *différance*, is not astructural; it produces systematic
> and regulated transformations which are able, at a certain point, to leave room
> for a structural science.[22]

Very well. Now try to imagine the structure of these transformations. Since we are in the "worst case scenario," where every piece of writing affects semantics, we can simply identify semantic transformations with writing that produces it. So we have to imagine the structure of "writing-in- general." Now, we can certainly imagine finite sequences of textual units being written out. We can also imagine infinite sequences of things being written out according to some finitary rule. Finally, let us add to this collection of texts the sequences that are only partially determinate: they proceed according to a finitary rule, where the rule is such that it leaves us some choice as to how to apply it. For example, the rule in some restaurants is to leave a tip of 15-20%,

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[22] Derrida, *Positions*, pp. 27-8.

so when you write a cheque to pay the bill you have the freedom to choose whether you will leave a 15% tip, 20% tip, 17.43% tip, or whatever. This does not determine your choice absolutely, and you can construct many different sequences according to this rule. We must take into account all such sequences. They are part of our everyday writing practices.

So, what is this collection of textual constructs? I do not know, but it looks a bit like Brouwer's construction of time continuum. According to Brouwer, our intuitive construction of time continuum can be modeled by choice-sequences (a concept he introduced in 1917). A choice sequence is a sequence of fractions construed according to a partially determinate rule similar to the tipping rule in restaurants. For instance, I could "define" a sequence of rational numbers by saying: the first element in the sequence is 1, while the (n+1)st element is any rational number X greater than the nth element of the sequence and such that $X^2 < 2$. Brouwer's continuum, the collection of these choice sequences, construed to take into account by its very construction the presently unknowable free choices, has unusual properties. Standard mathematical notions, such as the idea that if you have two real numbers X and Y then either X=Y, X<Y or X>Y, do not hold in Brouwer's continuum. In this type of mathematics, "binary logic," that is, the principle of the excluded middle, does not apply; there are no points or mathematical atoms in this structure, etc.

Brouwer's continuum is not an "object" in any currently accepted mathematical sense, because it is an act of consciousness, a construction that explicitly involves making free choices within the boundaries set by a rule. Brouwer argues that the continuum, although an intuitive construct, cannot be "known," cannot reduced to language or justified by logic. Nevertheless, the constriction of time continuum in consciousness is the ground of (mathematical) knowledge, or, as Husserl might put it, "the origin of geometry." And it does have structure: one can do mathematics on it, as Brouwer demonstrated.

Derrida, although his objective is to "deconstruct" idealist metaphysics and finally even the concept of the "origin" — and despite the fact he never mentions Brouwer anywhere — agrees that the construction of the continuity of time cannot ultimately be *known*. Our sense of it is not something we can justify in any scientifically acceptable manner. Therefore Husserl's attempt to delineate the intuitive construction of continuity of inner time, to give it a scientific description by means of phenomenological reflection seems bound to end up in trouble. To put it in a crudely simplified way, Husserl cannot jump outside of time, which he would have to do to ensure that the constitution of time consciousness is invariant in time. And if he cannot do that, then the ground of his "absolute science" remains only an Idea in the

Kantian sense. This is indeed the upshot of the critique of phenomenology Derrida outlined in the introduction to Husserl's *Origins of Geometry*:

> This thought unity, which makes the phenomenalization of time possible, is therefore always an Idea in the Kantian sense which never phenomenalizes itself. [...] This impotence and this impossibility are given in a primordial and pure consciousness of Difference. [23]

Brouwer spoke of the primordial intuition of the "falling apart of the life-moment" as being the fundamental phenomenon of conscious life (which, however, remains outside our powers of justification). Weyl called it "the medium of free becoming." And Derrida calls his version of it "the meaning of becoming in general," which cannot be mastered by any subject. Of course, the "construction" of the continuum as the set of semantic transformations incurred by unlimited writing — as I have interpreted it here — permits Derrida to avoid talking about time. He rather speaks of *Différance* as a kind of "spacing," which seems more natural in the case of writing.

So there are some important differences to keep in mind when drawing this analogy with Brouwer, not the least of which is the fact that Derrida does not consider *Différance* as the construct of a subject. However, there are further similarities between the two constructions, and this perhaps allows us to take some interpretive liberties for the sake of comparison. The most interesting feature of Brouwer's continuum — at least in the context of Derrida's as well as Heidegger's critique of the concept of *presence* — is that the continuum cannot be "separated," that is, we cannot pick out isolated points from it with atomic accuracy. Consequently, in this model of time continuum there is no "now," no "presence of the present" as Derrida might say. (Husserl also says something to the effect that "the present is not a point," but rather always involves a falling apart and reconstruction by means of retention and protention.) The reason for this is that Brouwer's time is fundamentally constructed by a "looking forward" into the unknown cloud of choices, existential possibilities that extend beyond the "present." Heidegger called this *Vorlaufen*, the "forward-directedness" of time. Let me digress here to explore this possible connection with Heidegger. It might be useful in understanding Derrida, whose work is undoubtedly related to Heidegger's.

Despite great differences in philosophical outlooks, there are points of agreement between Brouwer and Heidegger, notably in their critiques of logic and science and technology as a form of "man's domination by means of reason" — this is Brouwer's expression from the strange treatise *Life, Art and Mysticism* (1905), and it probably comes from Nietzsche — as well as in their opposition to the scientific/mathematical

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[23] Jacques Derrida, *Edmund Husserl's Origins of Geometry* (Stony Brook, Nicholas Hays, 1978), pp. 137, 153.

practices that proceed "inauthentically" by suppressing the practitioners' finitude and thus, idealizing their own ability, disregard their humanity, their "being for death" (as Heidegger would say). This concern with the "authentically" demonstrable was indeed one of the crucial features of Brouwer's attack on classical mathematics, which, I believe, Heidegger generalized and radicalized in *Being and Time*, along with working out a critique of idealism of which Brouwer was one of the most extreme mathematical representatives. At the very least, it seems natural to suppose that Heidegger would have been interested in the controversy over the time continuum that raged beside him as he was working on his *Being and Time*.

A small thought-experiment could help us understand how Heidegger could have utilized Brouwer's continuum in his critique of idealism. One of Heidegger's objectives seems to have been to demonstrate that the subject itself can in some sense derived from the structure of existential understanding. Suppose, for the sake of argument, that Brouwer's continuum is taken to represent that structure: not as a construct of the self-conscious subject but simply as a structure into which we are somehow "thrown." What would happen upon my being thrown into it? How would I see myself?

I could not say, I imagine, that I *am* this or that. Such an assertion would already involve me in claiming something about the present, and in Brouwer's continuum I cannot single out any point that could be called the present. On the contrary, time, the structure into which I am thrown, is construed by looking forward into the continuum of existential possibilities. Therefore, I would see myself not as I *am* but fundamentally as I *could be*. I am from the outset always *concerned* with the existential possibilities of my being: as soon as I find myself in (Brouwer's) time, I am necessarily projecting and my being automatically becomes an "issue of care." One way of describing this situation is to say that I understand myself as a being whose mode of being is to be concerned with its being.

This looks a bit like Heidegger's famously circular "definition" of *Dasein*, his derivation of the subject from the very structure of existential understanding, or, in this case, the structure of Brouwer's time continuum. With this derivation, Heidegger believes to have "displayed the birth certificate" of all concepts, idealist subject included, thus blowing away the entire idealist tradition, which he sees as a culmination of man's drive for mastery. This analogy, like all the others in this presentation, should not be understood as having any claim to exactness, as an attempt to "reduce" Heidegger's work to Brouwer's ideas about the constitution of time. I do believe, nevertheless, that it offers a question worth exploring, and poses a

more general problem of mutual relevance of Brouwer and Weyl on the one hand to Husserl and Heidegger on the other.

By the analogy between Brouwer's continuum and *Différance*, it is easy to imagine how Derrida could essentially rework Heidegger's derivation of the subject and claim, as he does, that "*Différance* produces the subject." A difference in the arguments would be that in Derrida's case one might speak of our being "thrown" into the text-in-general, into a world of ceaselessly transforming meanings. Also, it should be noted that Derrida is not uncritical of Heidegger. He goes a step further and criticizes Heidegger's "nostalgia for reappropriating the origin." Indeed, if Heidegger displayed "the origin of concepts," then this origin would also be the origin of his concept of the origin, which brings us into the realm of theological doctrines about the unmoved mover, and therefore right back to metaphysics. Derrida is highly critical of all metaphysics and hence also of Heidegger's claim to have "captured" the origin itself. He proposes that concept of the origin be abandoned:

> The origin was never constituted except reciprocally by a non-origin [*Différance*], which thus becomes the origin of the origin [...], which is to say that there is no absolute origin.[24]

In doing this, he is quite radical but also truer to the notion that (his reinvention of) Brouwer's continuum, *Différance*, does not permit talk about specific points, and therefore, in particular, about *the point of origin*, the point of beginning. In explaining the famous chapter heading from his book *Of Grammatology* — "The End of the Book and the Beginning of Writing" — he says that it should be understood as saying that "the book has no end and writing has no beginning."

It would be nice to know what Derrida's philosophy of mathematics might look like. He has written nothing about that. Yet his work poses serious questions to science. Is the effort directed at finding out what happened at "the beginning of the universe" simply an ill-posed problem? Perhaps, as Hawking has argued, the universe has no point of beginning, no "boundary" in a certain technical sense, in which case Derrida's critique of the notion of the origin could be relevant. Another question is whether it might be worth revisiting arguments of Brouwer, Weyl and Poincaré, which, as I believe and as I have tried to show here, might help reestablish communication between science and continental philosophy.

These meditations are, of course, highly speculative. I would not dare claim that they provide definite answers, or that they represent a "truth beyond reasonable doubt." But I do believe that it is better to be speculative than dismissive, and that mathematics, due to its complex historical entanglement with twentieth century

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[24] Jacques Derrida, *Of Grammatology* (Baltimore, Johns Hopkins University Press, 1976), p. 61.

thought, can be of assistance in rekindling a hermeneutical dialog between science and philosophy — a dialog more productive than the vitriolic exchange that flared up under the heading "Science Wars."

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