PRIMACY OF QUANTUM LOGIC IN THE NATURAL WORLD

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Abstract: This paper presents evidence from the fields of cognitive science and quantum information theory suggesting quantum theory to be the dominant fundamental logic in the natural world, in direct challenge to the long-held assumption that quantum logic only need be considered 'in the quantum realm.' A summary of the evolution of quantum logic and quantum theory is presented, along with an overview for the necessity of incomplete quantum knowledge, and some representative aspects of quantum logic. A case can be made that classical logic and theory is a subset of quantum logic and theory, given that elements of quantum physics exist that can never admit classical understanding, including: Bell's theorem, Hardy's theorem, and the Pusey-Barrett-Rudolph theorem. Support can be found for the primacy of quantum logic in the natural world in the cognitive sciences, where recent research studies recognize quantum logic in studies of: the subconscious, decisions involving unknown interconnected variables, memory, and question sequencing.

Keywords: Quantum theory; Quantum logic; Quantum cognition

EVOLUTION OF QUANTUM LOGIC

Logic provides a foundation for all branches of science through deductive, inductive, and abductive reasoning. American philosopher Charles Sanders Peirce proposed a model by which these three types of reasoning work together, starting with the goal of abduction being to find patterns in data and suggest plausible hypotheses; moving next to deduction which refines the hypothesis based upon realistic premises; and proceeding to induction to provide empirical substantiation. [1][2] Peirce pointed out that human minds have a natural abductive reasoning facility for making successful guesses and discerning meaning, writing, *"Mind is First, Matter is Second, Evolution is Third."* [3]

Discussions of quantum logic by logicians such as Woods and Peacock have focused on deductive quantum logic, which will be the primary focus of attention in this paper due to the current paucity of research in non-deductive reasoning. [4] Quantum logic is a relative newcomer in the field of logic, having arrived within the last hundred years with insights about the natural world that demand a change in the way we think about such things as nonlocality, causality, and consciousness. David Mermin's famous quote, *"Shut up and calculate!"* attained notoriety for expressing frustration at the humbling lack of clarity of quantum theory, compared with its reliably predictable mathematical results.

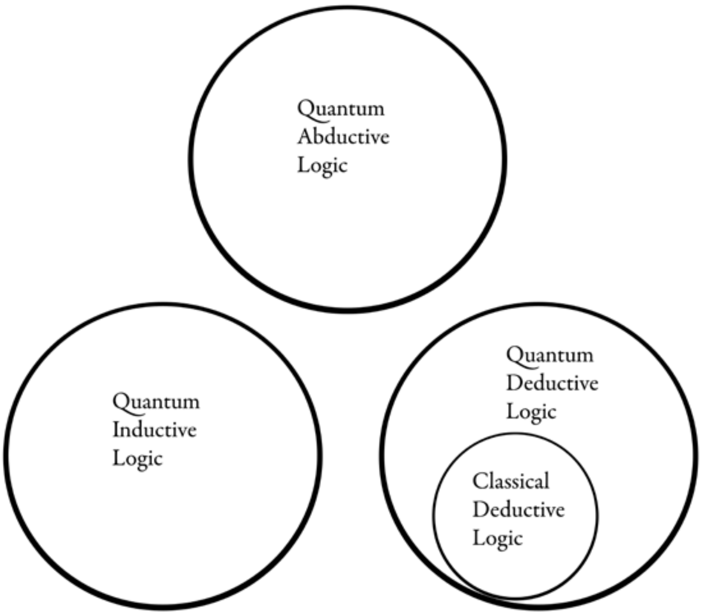

Figure 1. Relationship among three types of quantum logic; and quantum deductive logic and classical deductive logic

Just as Boolean logic provides the basis upon which modern classical computing logic is formed, clearly expressed quantum logic is now required in order to fully utilize quantum computing power. Boolean logic was developed in 1847 by English mathematician George Boole, who believed in what he called 'the process of analysis'—the process by which combinations of understandable symbols are attained. By interpreting conjunctions as intersections, disjunctions as unions, and negations as complements, Boolean logic provides the basis by which computing machines such as those invented by Charles Babbage can function, thanks to the way it sets forth a basis for symbolic language gates. Such logic gates give binary answers in the form of one or zero when assessing information reaching a basic logic gate such as AND, OR, and NOT, according to previously defined rules. While Boolean logic is excellent for consistently considering classes and propositions in orderly fashion, it cannot adequately address quantum theory.

Birkhoff and von Neumann initiated the quantum logic dialogue based on a more complicated phase-space, noting a correspondence between classical dynamics and Boolean logic. [5] The next fifty years ushered in a renaissance period for the logico-algebraic approach to quantum theory, inviting comparisons between intuitive logic and quantum logic [6]. The rise of quantum information now provides much richer and more comprehensive insights than classical theory can deliver. [7]

Quantum logic is new enough that there is not yet one comprehensive description of what it is and how it relates to other logic. Philosopher Edward Craig's assessment of the status of quantum logic in the "Routledge Encyclopedia of Philosophy" shows it is viewed differently by members of three camps: neoclassicists, quantum ontologists, and quantum logicians. [8] While neoclassicists consider quantum logic nothing more than a mathematical curiosity; quantum ontologists believe the word 'logic' is a misnomer when considered together with 'quantum,' since the physical world can be specified through probability measures. Quantum logicians hold the position that quantum logic is the most comprehensive deductive logic, emphasizing that attempts to fit quantum theory within classical logic and operations are doomed to fail. [9][10]

IMPROVED FOUNDATIONS FOR QUANTUM THEORY

Amidst the backdrop of lack of consensus regarding a full description of quantum logic, we find there exists additional uncertainty with regard to the way mathematics can best describe some of the attributes unique to quantum physics. In the standard quantum-probabilistic formalism developed by von Neumann, physical systems are described in terms of their association with Hilbert spaces, such that unit vectors of each system correspond to possible physical states. Quantum logical operations are decidedly different from classical Boolean operations. Von Neumann explains, "*… the relation between the properties of a physical system on the one hand, and the projections on the other, makes possible a sort of logical calculus with these. However, in contrast to the concepts of ordinary logic, this system is extended by the concept of 'simultaneous decidability' which is characteristic for quantum mechanics.*" [11]

Despite having developed the first formalism of quantum theory, Von Neumann expressed dissatisfaction with his mathematical formulation, remarking to Garret Birkhoff:

> "I would like to make a confession which may seem immoral: I do not believe absolutely in Hilbert space any more. After all, Hilbert space (as far as quantum mechanical things are concerned) was obtained by generalizing Euclidean space, footing on the principle of 'conserving the validity of all formal rules.' … Now we begin to believe that it is not the vectors which matter, but the lattice of all linear (closed subspaces) Because: (1) The vectors ought to represent physical states, but they do it redundantly, up to a complex factor, only (2) and besides, the states are merely a derived notion, the primitive (phenomenologically given) notion being the qualities which correspond to the linear closed subspaces." [12]

Von Neumann's concerns about the intrinsic inadequacies of Hilbert space are shared by members of the quantum foundations and quantum information theorist communities, as they work to find an improved foundation for quantum theory. Now that quantum information theorists have discovered quantum theory can be derived from simple axioms based on observations of laboratory operations, we are able to glean useful insights into the physical origin of the structure of quantum state spaces without confusing the epistemic map for the ontic territory. [13] Quantum information theorists have further shown quantum theory to be consistent with the notion of entanglement, while classical probability theory is not—suggesting that classical probability theory is a special case of quantum theory. [14]

APPRECIATING STATES OF INCOMPLETE KNOWLEDGE

When looking to physics to describe the nature of reality, a distinction is made between states of reality and states of knowledge, in which the *ontic state* is a state of reality, and the *epistemic state* is a state of knowledge. In classical physics, we study points in phase space, where we have complete specifications of all properties in the system, which are sometimes referred to as "Newtonian states." Such states are recognized to be *ontic* states. In consideration of classical statistical mechanics, probability distributions over phase spaces are also studied, with the realization that these descriptions do not describe all the properties of a system. These "Liouville states," as they are sometimes called, are *epistemic* states.

The distinction between ontic and epistemic states breaks down when considering epistemic states describing complete knowledge, since such special cases also contain a complete specification of a system's properties. For this reason, those taking the epistemic view of quantum states focus their attention on epistemic states describing incomplete knowledge. A key aspect of this viewpoint, as expressed by Spekkens, is that:

> *"all quantum states, mixed and pure, are states of incomplete knowledge."* [15]

One of the most intriguing aspects of examining Liouville states with quantum states is the striking similarity between phenomena exhibited in Liouville mechanics and what is observed in pure quantum states that are not seen in systems involving states of complete knowledge. These phenomena include: a no-cloning theorem, impossibility of discriminating such states with absolute certainty, lack of exponential divergence between such states under chaotic evolution, and entanglement. As Spekkens observes,

> "This suggests that one would obtain a better analogy with quantum theory if states of complete knowledge were somehow impossible to achieve, that is, if somehow maximal knowledge was always incomplete knowledge… … In fact, the toy theory suggests that the restriction on knowledge should take a particular form, namely, that one's knowledge be quantitatively equal to one's ignorance in a state of maximal knowledge." [15]

Spekkens draws our attention to contextuality, and the idea that whereas our choice of how we conduct a given experiment does not affect the experimental statistical results, that choice definitely influences knowledge about what is going on in reality. And while we might have expected that we could detect such changes experimentally, this very notion of contextuality that Spekkens presents shows us that such informational signals do not get through. When we consider reality being nonlocal, then adding this idea of contextuality indicates that exceptional fine-tuning must be operating to prevent our changes in knowledge from influencing changes in predictions of what we will observe.

Systems featuring incomplete knowledge embody special qualities, and quantum logic has the edge when it comes to providing insights into learning the relationship between knowledge, space, and time.

INSIGHTS FROM DERIVING QUANTUM THEORY

Many seemingly intractable problems with the orthodox interpretation might primarily be an artifact of viewing quantum theory through classical logic assumptions. In order to minimize such classical bias, some quantum foundations physicists found a way to derive quantum theory from scratch. In 2001, Lucien Hardy presented an elegant method for deriving quantum theory from five simple axioms involving: probabilities, simplicity, subspaces, composite systems, and continuity. [16] Masanes and Mueller continued this approach, pointing out in their 2011 paper that derivation of quantum theory from five simple physical requirements *"is more similar to the usual formulation of special relativity, where two simple physical requirements—the principles of relativity and light speed invariance—are used to derive the mathematical structure of Minkowski space-time."* [13]

Careful review of quantum phenomena that can't be described in classical terms can provide unique insights into the true ontic reality of Nature, as well as practical advantages for Information Science. Giulio Chiribella's contribution is inclusion of a sixth axiom—the assumption of a purification postulate—such that Schrodinger's assessment of entanglement provides the essence of the postulate thus,

"Maximal knowledge of a total system does not necessarily include maximal knowledge of all its parts." [17]

Chiribella points out that theories can only satisfy this purification postulate by containing entangled states. By combining this postulate with the five axioms, Chiribella *et al.* demonstrated it is possible to successfully derive all of quantum theory. [14]

In addition to the conceptual clarity conferred by this derivation of quantum theory, it's clear that many quantum phenomena that seem mysterious from an ontic viewpoint appear to be much more natural when viewed from an epistemic perspective, where we consider quantum theory to be a kind of nonclassical probability theory. Such phenomena as interference, entanglement, and teleportation can be recognized as making sense not just mathematically—but also from an epistemic interpretation. Much confusion is cleared up when viewing the quantum state as a probability distribution occurring in an imaginal realm, rather than a physical state of reality. From such a standpoint, the 'measurement problem' can be viewed as being more of a problem with the orthodox interpretation of quantum theory than an intrinsic aspect of quantum theory. This occurs due to the fact that in the orthodox interpretation of quantum theory a quantum state evolves, is measured at some point in time, and collapses—giving the appearance of being time-asymmetric. In contrast, the psi-epistemic quantum information theory perspective considers retrocausal influences to be a natural part of the quantum process. [15]

SOME ASPECTS OF QUANTUM THEORY CAN NEVER BE CLASSICAL

The fact that some aspects of quantum physics can never admit a classical understanding establishes a strong case for Nature being fundamentally quantum. Three aspects of quantum physics that do not logically fit within classical constructs include: Bell's theorem—demonstrating how no physical theory of local or hidden variables can ever reproduce all the predictions of quantum mechanics, Hardy's theorem—showing that even finite dimensional quantum systems must contain an infinite amount of information, and the Pusey-Barrett-Rudolph theorem—indicating that the wave function must be an objective property of an individual quantum system. [18][19][20]

Irish physicist John Stewart Bell's theorem presents one of the strongest proofs of quantum non-locality. Bell considered the Einstein Podolsky Rosen (EPR) system, and proved that all conceivable models of reality must incorporate this instant connection—showing that despite the fact that relativity prohibits instantaneous connections, the reality of the EPR particles is such that their initial contact must create an instantaneous link between them. [21]

Lucien Hardy designed an experiment in which an electron and its antiparticle, a positron, may be detected in one of two interferometers. However, a certain combination of detectors can only be selected by the pair if the two particles have previously traveled along bent path trajectories in which they annihilated each other—which means they can't reach the detectors. Except that in many cases, they *do* reach the detectors. Hardy's Theorem shows that even finite dimensional quantum systems must contain an infinite amount of information—and as other physicists have pointed out, antimatter is not required to demonstrate the success of these experiments. [18][19]

QUANTUM MODELS OF COGNITION

Support can be found for the primacy of quantum logic in the natural world in the cognitive sciences.The connection between logic and cognition is strong, since the original purpose of logic is to reveal the structure of human reasoning. While a bias toward reductionist materialism and Boolean logic in the field of cognitive science has contributed a great deal in terms of understanding cognitive mechanisms, many problems remain unsolved. The new field of quantum cognition has presented a common set of principles from quantum theory that explain some baffling behavioral phenomena observed in human decision-making. Quantum theory provides possible explanations for: "irrational" decision making, conjunction and disjunction probability judgment errors, over and under-extension errors in conceptual combinations, ambiguous concepts, order effects on probabilistic interference, interference of categorization on decision making, attitude question order effects, and a variety of other surprising results from the field of decision research. [22][23]

A compelling case has recently been made by researchers who have systematically compared classical and quantum probability theories while modeling cognitive phenomena. They note that with regard to their value in successfully modeling human cognition, quantum and classical probability theory often exhibit *"perfect agreement when all the events under consideration are compatible. The need for the quantum approach only arises when incompatible events are involved, which necessarily imposes a sequential evaluation of the events. This incompatibility produces superposition states of uncertainty that result in violations of some of the important laws of classical probability theory."* [24]

One of the advantages of a human cognition model based on quantum probability is that such a model accounts for what is observed in human cognitive behavior. Choices people actually make in the Prisoner's Dilemma and two-stage gambling decisions are better explained by quantum probability than classical, and such things as nonverbal cognition and memory are better understood with quantum theory and logic as well.

Two aspects of quantum theory that are especially promising for providing a strong framework for addressing long-standing problems in the field of cognition are: contextuality and entanglement. [6][25] Contextuality can be understood through the somewhat analogous concept of interference, which can occur in superposed quantum systems; ie: we know the meaning of a word based on its surrounding context.

Cognitive scientists and linguists note that quantum logic matches human behavior, and that much of our thinking operates on a largely unconscious level. [26] Gärdenfors *et al* advises that we must go beneath the symbolic level of cognition, pointing out that, *"… information about an object may be of two kinds: propositional and conceptual. When the new information is propositional, one learns new facts about the object, for example, that x is a penguin. When the new information is conceptual, one categorizes the object in a new way, for example, x is seen as a penguin instead of just as a bird."* [27].

A 'quantum logic of down below' is evidenced in a variety of cognitive functions where the strength of associations between concepts changes dynamically under the influence of context, which then influences the defaults harbored within symbolic levels of cognition. [28] Human memory thus appears to operate through quantum information retrieval, [29] and people respond to survey question sequencing according to quantum probability theory, providing a simple account for surprising regularity regarding measurement order effects. [30]

Examination of symbolic level information retrieval—such as what is required, for example, to determine the correct context of a word like "bat"—prompts us to observe that human memory appears to operate via quantum information retrieval processes. We can determine what kind of "bat" is being referred to, thanks to recognizing words commonly associated with it, such as either "winged, flying, mammal" or "baseball, homerun, team." Quantum entanglement provides a conceptual basis by which seemingly separated quantum systems behave as one, and contextual meaning can be correctly understood. [29][31]

Figure 2. Illustrative contextual word clusters associated with the word "bat"

A key concept in the field of quantum cognition is recognition of the fact that humans answer the exact same questions differently, depending on the order of the questions being asked. This can make a big difference when the effect is spread across a large number of people who are voting or being surveyed for their opinions. The idea of "quantum question" equality is based on the starting assumption that a person's knowledge used for answering questions can be represented in the form of a very high multidimensional space, which can be described by a set of orthogonal axes. While this representational arrangement of a person's knowledge does not change based on questions or the context surrounding questions, the way the knowledge is utilized does change based on both the questions asked and the context surrounding the questions. The so-called "quantum question" equality presented by Wang *et al* shows how the very same quantum probability theory that explain otherwise mysterious noncommutativity of measurements in physics, can also provide excellent measurement predictions for question order effects in social and behavioral science experiments.

"(i) Human judgments, such as attitude judgments, are often not simply read out from memory, but rather, they are constructed from the cognitive state for the question at hand; and (ii) drawing a conclusion from one question changes the context and disturbs the cognitive system, which then (iii) affects the answer to the next question, producing order effects, so that (iv) human judgments do not always obey the commutative rule of Boolean logic. If we replace "human judgments" with "physical measurements" and replace "cognitive system" with "physical system," then these are exactly the same reasons that led physicists to develop quantum theory in the first place." [30]

Entanglement is also evidenced in experiments involving choices when playing games. Study participants were informed that they had just gambled in a situation in which they had even odds of either winning $200 or losing $100, and were then asked to choose whether they would like to play the same gamble a second time. In one condition, participants were informed they'd won their first play; in a second condition, they were told they'd lost the first play; and in a third condition, they were not provided with any information regarding the outcome. When people were given an opportunity to play this two-stage gambling choice game twice in a row, participants surprisingly chose to play again when they learned the result of their first play (69% win / 59% loss) compared with choosing to play again with initial results unknown only 36% of the time, and these results were observed when real money was at stake. [32]

There is a noticeable difference between the commutative axiom of classical logic, and the complementary quality of quantum logic. Operations can be considered in any sequence in classical deductive logical operations, so the order of considering two separate propositions such as "one prisoner is guilty" and "a prisoner should be punished" can be accepted or rejected when considered in any order. Quantum theory requires sequential consideration, due to contextual relationships between ideas of guilt and punishment. [33]

Quantum cognition researchers consider the set of basis states as a set of preference orders over actions, allowing for individuals to experience superpositions of all the possible orders, remaining uncommitted to any particular order. Through such research methods, researchers have thus found evidence of quantum processes at work in human cognition. If human behavior followed the logic of a Markov process, people would be committed to one and only one preference order at any given point in time—though that preference order may change from time to time. An individual following the logic of a quantum process, on the other hand, experiences a superposition of preference orders, so at any given point in time they will report being uncommitted to any particular order. [32]

TO SNITCH OR NOT TO SNITCH

Similar unexpected results were obtained in two-person Prisoner's Dilemma games, violating notions of rational reasoning put forth in Savage's "sure thing" principle. [34] Savage's "sure-thing" principle states that if a decision maker would take a certain action if he knew that a particular event E occurred, and also if he knew that the negation of event E occurred, then he would take that action even if he knows nothing about event E. Human reasoning in the prisoner's dilemma has proven to be a better match for quantum logic rather than classical logic. In the prisoner's dilemma, two suspects, Alice and Bob, are apprehended and interrogated separately in isolation from one another. Both Alice and Bob are given the same deal that if they both remain silent (cooperate) and don't snitch on the other (defect), they'll both go free. If Bob snitches on Alice, but Alice does not snitch on Bob, then Bob goes free and receives a reward, while Alice receives the maximum sentence. The same scenario unfolds if Alice snitches and Bob does not. If both snitch on each other, they both receive reduced sentences. According to classical logic, each person's self-interest is expected to drive their decision that each can be expected to snitch on the other 90% of the time. Actual tests on human subjects have repeatedly shown that people choose not to snitch 40% of the time, rather than the classically predicted 10%.

Intriguingly, when one calculates the odds according to quantum probability theory, a very different strategy emerges—one that appears to be part of intrinsic human reasoning. In experimental studies, people favor envisioning the optimal outcome in which both Alice and Bob remain silent, and both go free. When we assume a condition of "maximal entanglement" in assessing the quantum probabilities for this experiment, we find that the most rational quantum mathematical choice is for both to remain silent. [35]

Subsequent studies have replicated these results, providing empirical findings suggesting that classical probability theory is not an appropriate framework for modeling cognition. Researchers in the field of quantum cognition believe that human cognition deserves to be modeled probabilistically, while pointing out that, "*classical probability theory is too restrictive to fully describe human cognition.*" [36]

CONCLUSIONS

While we have long presumed quantum logic to operate either alongside or within the classical realm, we stand to benefit from contemplating classical theory and physics as a special case within the bigger quantum reality. This paper finds support for the view of the quantum logicians who assert quantum logic to be the most comprehensive deductive logic. This is demonstrated by the derivability of quantum theory from one additional fundamental axiom beyond that required for deriving classical theory, and evidence that some fundamental aspects of quantum physics can never admit a classical understanding. Recognition that classical theory and logic is a 'special case' subset of the greater quantum whole invites us to completely reassess our assumptions regarding the way we view the world and what we consider to be 'logical.'

The fundamental nature of quantum logic presents us with an opportunity by which many previously mysterious natural phenomena that are exclusively part of quantum theory can now be better understood, such as entanglement. This paper also finds support for a connection between the apparent primacy of quantum deductive logic in physics, and the importance of quantum reasoning in human cognitive processes. Consideration of quantum theory and logic being primary in the natural world helps explain why human evolution features a reasoning system based on quantum probability, and reveals quantum logic in the way humans make decisions and record memories.

Better understanding of quantum logic suggests we can investigate how we communicate contextuality with others in our social networks, how we can sense possible realities amidst a superposition of states, and how cognitive states are entangled. Further research into the fields of inductive and abductive quantum reasoning is needed in order to determine what insights can be found in non-deductive reasoning to provide a fuller understanding of such areas as human cognitive functioning, quantum computing, and artificial intelligence.

When we gain a deeper appreciation for the possibility that quantum logic is primary in the natural world, we see how humanity stands to benefit from embracing the innate quantum logic implicit in everything. We can thus envision how the addition of quantum theory ushers in a new view of all areas of study, including: biology, psychology, sociology, cosmology, statistics, and history. The idea that quantum phenomena occur at all levels—not merely at microscopic quantum levels—indicates we are able to develop a more functionally predictive and naturally based quantum perspective that promises to completely revise our worldview.

One aspect of this new worldview appears to be that unifying all logic under one quantum umbrella comes at a price. Despite our desire to one day know all there is to know, quantum theory now informs us that in a state of maximal knowledge, one's knowledge is quantitatively equal to one's ignorance. It is possible to find a sense of awe and reverence as we appreciate a side of Nature described in Lao Tzu's observation, *"the more you know, the less you understand."*

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