LOGIC(S) OF THE VALUE FORM

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ABSTRACT: Alain Badiou has grounded the axiom of materialist dialectic on the following proposition: the Three supplements the reality of the Two. Consequently, if we are to call Badiou's *ouvre* a truly dialectical body of philosophy, it should be capable of congruence with its forerunning counterparts: Hegel's and Marx's dialectics. The paper aims to provide a proof by analogy between Badiou's materialist or affirmative dialectic and Marx's dialectics of the value form using rudimentary category theory. In order to sustain the converging pathway, the paper presupposes a synthesis of specific conceptions in the two strands of contemporary Marxian thought. Namely, structuralism of Louis Althusser, with a particular reference to Jacques Rancière's contribution in *Reading Capital*, and secondly, German »Neue Marx Lektüre«, new reading of Marx, initiated by Adorno's students Hans Georg Backhaus, Helmut Reichelt and Alfred Schmidt. The paper implicitly reintroduces the concepts of »structural causality« and »absent cause« as a specific algebraic property in the structure of social exchange, intertwined with the topological adherence of object-moments of money commodity - an analogous category-theory approach to the one developed by Alain Badiou in his work *Logics of Worlds*.

KEYWORDS: Value form; Badiou; Marx; Category theory; New reading of Marx; Structural Marxism

The social exchange of products, with the addition of money or other various types of contracts (futures, debt, etc.), has been a feature of human kind for millennia. When examining history, one encounters a manifold of different worldly configurations – frameworks –, but as is the case for us, we must proceed in our endeavour to demystify the logics of our current world – the capitalist institutional framework. Marx pursued this mission by critically examining the twofold nature of labour, corresponding to the twofold nature of the commodity, one that presides in this particular framework. On one hand, this framework entails the social division of labour, while on the other, it

posits the conditions for commodity production and wealth formation, known by the name – the capitalist mode of production. These types of societies introduce the *commodity* as a basic cell of exchange, a useful and relational value-objectivity [*Wertgegenständlichkeit*], i.e. the material bearer of exchange value. The social materiality of exchange is *in actu* a social division of different concrete labours including their results – the finished products of labour. Marx introduces the *value-form* in order to show how this double character of products manifests itself in commercial intercourse; entering exchange, the products in their natural forms as use values simultaneously become the bearers of (exchange) value. Positing *value form* thus becomes crucial; the sensuous objectivity character of commodities as physical objects, once put in relation with one another, acquire an additional, immediately ungraspable property – that they are, in fact, all products of the same social substance, human labour. Whenceforth it becomes clear, that we are dealing with a relation between sensuous objectivity of a thing and value-objectivity deriving from a social constellation. To posit such a relation, one needs the presupposition of at least two commodities standing side-by-side. On the other hand, precisely this positing sets forth the conditions for the existence of its completed form – the money form – one that appears as something transhistorically given and needing no "scientific" treatment.

Marx's novel scientific approach was to introduce the critique of categories inherent in political economy. He engaged in the subversive task of probing the most concrete and complexly developed forms to their innermost abstract cores and sources. It has to be emphasised that the whole of his analytical work takes place inside the capitalist mode of production – the capitalist institutional framework –, where the *regime of value form* resides. This regime entails a specific structuration of social relations, i.e. a relational framework between objects. We will proceed with our analysis of economic objectivity by introducing the grounds for *the dialectics of the value form* thought as a *fundamental regime, pertaining to logics of appearing of economic objects and their immanent relations.* We will add to the contributions of Mario L. Robles-Báez, Christopher J. Arthur, and others[1] in their assessments of formal logic inherent in the unfolding of forms from Simple, Accidental to the Money Form of Value. We will reconstruct the developmental stages of the value form, from simple to money form, on the basis of a mathematical branch called category theory, operating on categories and multiplicities

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[1]Mario L. Robles-Báez, 'On Marx's Dialectic of the Genesis of the Money Form', *International Journal of Political Economy*, vol. 27, issue 3, 1997, pp. 35-64;
Christopher J. Arthur, 'Money and the Form of Value', in Ricardo Bellofiore and Nicola Taylor (eds.), *The Constitution of Capital – Essays on Volume I of Marx's Capital*, London, Palgrave Macmillan, 2004.;
'Marx, Hegel and the Value-form', in Fred Moseley and Tony Smith (eds.), *Marx's Capital and Hegel's Logic: a reexamination.* Leiden, Brill, 2014.

of sets in an axiomatic fashion. Following these steps, which are innovatively used by Alain Badiou in his *Logics of Worlds*, we will seek to define the transcendental operation of appearing through a logical analogy of these two dialectical operations.

VALUE FORM REVISITED

In *Capital*, Marx begins his analysis of commodity following the steps of Hegelian dialectical logic: with a scission into qualitative and quantitative determinations of a commodity. Every commodity is *firstly* determined qualitatively; its natural properties, which satisfy particular needs, attribute to it use value. If we progress further into the nature of things as useful objects, we *secondly* find that their quantitative determinations (Marx speaks about yards of linen, hours of labour-time, bushels of wheat or pounds of gold) form a unity with qualitative determinations and connect to a material content of wealth, i.e. a social form of (exchange) value. In order for the exchange of different commodities to take place, an equilibration of their determinate quantum to a common denominator needs to be established, something that makes a convenient value comparison of two commodities, while they themselves are measured to a completely different singular quantum. Aside from possessing its material and sensuous properties that makes it a use-objectivity, a thing, according to Marx, possesses one more property that is common to all others: *human labour*. Completing the abstraction of all sensuous properties for a commodity enables Marx to posit value as a social relation, devoid of a usefulness predicate, i.e. a subtraction of all concrete and useful aspects of labour leading towards the concept of *abstract human labour*. It is precisely this coagulation that constitutes value as "crystallization of human labour in general", an expression in the same unity, – value – as "crystals of common social substance". The objectification of labour becomes the quantum of expended labour-power, inducing the measure of value: "What exclusively determines the magnitude of the value of any article is therefore the amount of labour socially necessary, or the labour-time socially necessary for its production."[2] The next task is of course: "Now we know the substance of value. It is labour. We know its unit of measurement. It is labour-time. We have yet to analyse its form, which precisely stamps the value as an exchange-value."[3]

Let us briefly overview Marx's development of value form and provide some additional comments about the previously mentioned authors. First, **the Simple, Isolated, or Accidental Form of Value:**

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[2] Karl Marx, *Capital: A Critique of Political Economy, Volume One*, trans. Ben Fowkes, London, Penguin Books, 1993, pp. 129.

[3] Karl Marx, 'The Commodity. Chapter One, Volume One, of the first edition of Capital', in: Dragstedt, A. (ed. and Translator), *Value: Studies By Karl Marx*, London, New Park Publications, 1976, pp. 11.

Relative value-form   **Equivalent form**

x commodity A   =   y commodity B

x commodity A is worth y commodity B

Marx poses both sides of this expression as a contradiction – as inseparable, but mutually exclusive poles. Since commodity A stands in the relative value-form, it "uses" the bodily or natural form of commodity B (the value of e.g. linen is expressed with a coat), standing in equivalent form in order to achieve its social account (value relation). If we were to say on the contrary that 20 yards of linen = 20 yards of linen, we could not speak of any value expressive relation, for there would be no reflection of social relation. While value is presented with the operator *is worth*, it presupposes opposite poles and consequently violates formal reflexivity determination of (exchange) value. Henceforth, exchange value cannot be derived from commodity itself, for the latter stands in-itself only as use value. What is crucial here is the positioning of both commodities in the value expression, where as far as dialectical contradiction goes, value form cancels the symmetry of expression. So, $aRb \neq bRa$, or 20 yards of linen is worth 1 coat and 1 coat is worth 20 yards of linen do not represent the same value expressions. We are dealing with a specific objectivity, a relational mediation between two objects of commodity form resting upon dialectical logic. An inverse operation thus means exclusion – Marx says in the appendix to the first edition of *Capital*: "As regards the form they are not only different but opposed. […] Because they have changed their respective positions in the same expression of value, they have changed value-form."[4] In this instance, Arthur speaks of *reversed relation*[5] instead of converse relation, which implies $aRb = bRa$. So, one must be precise in operating with the equation x commodity A = y commodity B and the expression x commodity A is worth y commodity B, and its reverse:

> Marx then moves to consider the *reversed* expression (in his terms '*y* commodity *B* = *x* commodity *A*'), in our terms, '*yB* expresses its value in the use-value *zA*'. It is important to consider whether the reverse expression is the *same* as the first. Using ' = ' it is of course thus symmetrical. But, as we have seen, Marx contradicts this implication of ' = ', and so he does here, strongly emphasizing that *a different expression* of value is evolved. Marx is quite correct. Although '*yB* expresses its

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[4] Karl Marx, 'The Value-Form', *Capital and Class* 4, Spring 1978, pp. 135.

[5] *Cf.* Christopher J. Arthur, 'Money and the Form of Value', in Ricardo Bellofiore and Nicola Taylor, (eds.), *The Constitution of Capital – Essays on Volume I of Marx's Capital*, London, Palgrave Macmillan, 2004,p p. 39.

value in the use-value *zA*' looks similar to '*zA* expresses its value in the use-value *yB*' it is a different case because it is not derivable from the latter as its converse. (Rather, the converse is 'The use-value *yB* expresses the value of *zA*'.)[6]

Solving the presentation of reflexivity and reversed symmetry of the simple form of value, Marx continues with the problems of transitivity in **the Expanded Form of Value:**

$$z \text{ of commodity A expresses its value in} \begin{cases} u \text{ commodity B} \\ \quad \textbf{or} \\ v \text{ commodity C} \\ \quad \textbf{or} \\ w \text{ commodity D} \\ \quad \textbf{or} \\ y \text{ commodity E} \\ \quad etc. \end{cases}$$

or as Marx expresses it in the first edition of *Capital*, a series of Simple forms emerge:

x commodity A = a commodity B or,	f commodity B = g commodity or,	
x commodity A = b commodity C or,	f commodity B = h commodity C or,	...
x commodity A = c commodity D or,	f commodity B = i commodity D or,	
etc.	etc.	

By putting the expanded form side by side, we want to highlight certain aspects of the expanded commodity universe. In such a way, innumerable commodities come to mirror linen's value, but as far as the former expression goes, it does not necessitate the existence of N-different expanded forms. Every set of one-commodity expressing its value in all others reciprocally excludes itself from standing in the equivalent form of value. Aside from this corollary, every N commodity conducts N-1 value expressing relations and by doing so polarly excludes their (all assembly of N-1 commodities)

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[6] *Ibid.* pp. 47.

capability of standing in the relative form of value. Marx condenses the problems associated with the expanded value form into three defects: an infinitely open chain of commodities, a motely mosaic of disparate and unconnected expressions of value, and each commodity in relative form enters in a series of endless expressions of value. What needs to be emphasised is the following: although we encounter multiplicities of commodities, e.g. A, B, C… Z, the perspective of exchange is always subjacent to the agent of exchange; the first trader wants to exchange commodity A, the second commodity B, and so forth. Only a particular commodity of an individual counts as an object of exchange. This implies a negation of the *simultaneous conjunctive existence* of all expanded forms of value. What we are dealing with here is its obverse, an *exclusive disjunction*, hence the operator *or* ($\vee$) in the presentation of expanded forms. On the other hand, this operator breaks any chance of transitivity for the value form, since it ceaselessly and exclusively expresses value of *one* commodity against all others, but it cannot do so simultaneously. Any contingent set of commodities (A, B, C, etc.) in the expanded form reciprocally excludes the *one* commodity in relative form from the *multiplicity* of commodities in the equivalent form. At this point, we are positing the presupposition of a constitutive *surplus moment* of an object, being that of a general equivalent, leading to general value form. In its expanded form, this moment is barred because the commodity crystallisation of value appearance is still incomplete, lost in the "jelly" multiplicity of simple commodity forms. Many owners of commodities trade their concrete merchandise with others, bringing forth a totality of exchange and achieving a converse (not symmetrical) relation of the **General Form of Value**, which is a simple form (one commodity in equivalent form) and a unified form (same body of a commodity each time) that comes to expresses value. Many different commodities are exchanged against one and *the same* commodity, e.g. linen.

$$
\left.
\begin{array}{ll}
u\ commodity\ B & = \\
v\ commodity\ C & = \\
w\ commodity\ D & = \\
y\ commodity E & = \\
\quad etc.
\end{array}
\right\} \quad = \quad z\ commodity\ \text{A}
$$

Born is the real-abstraction, a sensuous-suprasensuous thing – value in the world of commodities. The expanded form does not lead to this; it would only amount to the numerous twisting of particular expanded forms and the exclusion of others, which in the long run would lead to the infinite regress of mutual exclusions, failing to introduce a universal equivalent. An important consequence of the expanded form is the following relation: a single one commodity in the relative form and a multiplicity of

commodities in the equivalent form. Whilst in objectivity we are always faced with the simple form, we shall slightly modify the interpretation and say that we are confronted with *one* commodity and multiple commodities *counted-as-one*. The gateway from expanded to general form is determined through the unification of two previous form moments and mediated with the power of dialectical unfolding.[7] This unification can in formal logic be understood as a transformation of the logical operator *or* to *and*, referring to the secured existence of a third term – namely, value –, to which we can now subsume the commensurability of two commodities. A proof can be conveyed by reversing an expanded form: in that case we would get a myriad of different universal equivalents. Conversely, the general form adds a qualitative operation of abstraction surmounting to value-independence, its embodiment, the existence of the "Animal" in the kingdom of concrete animals, lions, tigers, rabbits, etc.

A change can occur if all the *or* ($\vee$) operators of the expanded form are simultaneously transformed into *and* ($\wedge$), or alternatively, one after the other are successively overturned. The latter would induce a slow, but progressive expunging of commodities in the equilibrium form with the same outcome. To give an example: suppose we define the world of commodities as composed of N commodities; in that case, we have $N(N-1)$ variations of expanded commodity forms. It follows that we have N commodities in relative form and $N-1$ in equivalent form. We proceed with the successive iteration from $N-1$ in relative form and $N-2$ in the equilibrium form … to $N-(N-1)$ in relative and 1 in the equilibrium form. The successive excluding of commodities one-by-one would leave us with the one remaining commodity[8] – *in fine* a universal-concrete Equivalent Commodity. Marx also distinguishes the expanded and general form of value according to the process of exclusion:

This is that in the expanded value-form (form II) one commodity excludes all the

[7] *Cf.* Christopher J. Arthur, 'Marx, Hegel and the Value-form' and Abelardo Mariña-Flores and Mario L. Robles-Báez, *A Critique of Benetti and Cartelier's Critical Examination of Marx's Theory of Money*, accessible at*: http://copejournal.com/wp-content/uploads/2015/12/Marina-Flores-and-Robles-B%C3%A1ez-A-Critique-of-Benetti-and-Carteliers-Critical-Examination-of-Marxs-Theory-of-Money-1998.pdf*
All authors in their ways summon the active drops of hegelian dialectic at work in Marx's shift from expanded to general value form. Mariña-Flores and Robles-Báez explicitly recall Hegel's section on "*Repulsion and Attraction*" in the *Science of Logic*, while Arthur adds a complementary reading in the Book two "*The Relation of Force and its Expression*". They understand the shift from expanded to general form in terms of dialectical twist (and not formal), meaning an asymmetric reciprocal exclusion. The fact that poles of value form are not symmetrically interchangeable, or that a multiplicity of simultaneous simple forms cannot arrive to general form, so that this inverse [*verkehrten*] of form – one and count-as-one - "produces" content-wise a new form determined on value. Particular-Universal-Singular.
[8] We presuppose N, N-1 … N-n and not N, $(N-2)/2$… since the relations are mutually exclusive; $aRb \neq bRa$

others in order to express its own value in them. This exclusion can be a purely subjective process, e.g. a process traced out by the possessor of linen (*z.B. ein Prozess des Leinwandbesitzers*) who assesses the value of his own commodity in many other commodities As opposed to this a commodity is in general equivalent-form (form III) only because and insofar as it itself is excluded as equivalent by all other commodities. The exclusion is here an objective (*objektiver*) process independent of the excluded commodity.[9]

A General Equivalent is a result of objective social process, pertaining to the formation of "*universal body-value*", "*universal materialization of abstract human labour*" – valorisation [*Verwertung*]. From here on, the notion of labour-in-general comes into full light; by opposing the products of two different concrete labours a relation between labour and value emerges, mediated by the general (equivalent). Such a relation has for its condition the finished process of reciprocal exclusion of all commodities, except the *One* against *many-counted-as-one*. In effect the One is ousted from standing in the relative form. Although the objective social process runs on the back side of value constitution and abstract labour, the question of money commodity remains open, since the universal equivalent in itself does not yet possess the functionalities of money. Marx had cunningly separated the production of *object-surplus moment* – universal equivalent – into *objectivity* of social exchange and the *subjective* determination of specific commodity as money-object. It is not until Chapter 2 of *Capital Vol. I* that he explicates the scission:

> In their difficulties our commodity-owners think like Faust: 'In the beginning was the deed.'. They have therefore already acted before thinking. The natural laws of the commodity have manifested themselves in the natural instinct of the owners of commodities. They can only bring their commodities into relation as values, and therefore as commodities, by bringing them into an opposing relation with some one other commodity, which serves as the universal equivalent. We have already reached that result by our analysis of the commodity. But only *the action of society* [*gesellschaftliche Tat*] can turn a particular commodity into the universal equivalent. […] It thus becomes – money.[10]

The transition from universal equivalent to money is not merely a direct deduction of parity; the universal equivalent is the outcome of an objective social process presented with the logical unfolding of the value form, and as such is primarily something *undecidable* (any commodity can attain the universal equivalent property), hence it needs a *subjective verification* of the process in order to attest the generality residing in the actuality of exchange relations. To capture this, Marx finishes with the fourth, **the**

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[9] Karl Marx, "The Value-Form", p. 148-149.

[10] Karl Marx, *Capital: Vol. I,* p. 180-181.

Money Form:

$$
\left.
\begin{array}{l}
u\ commodity\ B\ = \\
v\ commodity\ C\ = \\
w\ commodity\ D\ = \\
y\ commodity\ E\ = \\
z\ commodity\ A\ = \\
\qquad etc.
\end{array}
\right\} \ = \ 2\ \text{ounces of gold}
$$

Why and how do we get from the general form of value to the money form? Logically speaking, both forms are the same; the difference is only in the equivalent form – from universal equivalent to money. This is the result of a subjective process. A choice is involved as to which commodity among all indiscernible others will have been chosen to stand-in as money. What is in fact done is the attribution of an additional property to a specific commodity, discerning it from all the rest in the world of commodities. The following question arises: In what manner is it possible to think the immanence of formal relation between different objects in the value form?

OBJECTS, RELATIONS, CATEGORIES – MONEY

Let us begin our analysis of value form and its immanent object relations with a longer passage by Helmut Brentel talking about the constitution of the economic-social forms of bourgeois society:

> As a critique of political economy, as a critique of its categorical development is what Marx renders to put forward in a theory: the constitution of economic-social forms of bourgeois society. Economic objectivity *qua* "value-objectivity" is social [*gesellschaftliche*] form – the specific social [*soziale*] form of labour – in a twofold sense: as thing and as relation simultaneously. In the latter sense (as relation): the relation of equality between different labours is constituted as the specifically social form of the sociality of labours (Social form I). In the former sense (as thing): on the one hand, value itself gains an objective character and is manifested as the fetishistic natural feature of the products of labour in bourgeois society (Form I); on the other hand, such value must attain a form of existence in a value-form, in the natural form of another commodity as an equivalent form (Social form II). The distinguished object of bourgeois society, value-objectivity, has social *object*-characteristics and exists only as a social relation – in the end as a

processual subsistence through valorising process of Capital.[11]

The social form of labour has an economic objectivity that is comprised of a double existence: as a *thing* and as a *relation* simultaneously. What is at stake is the dialectical relation mediating things or, put another way, a relation between objects of exchange. We are confronted with *Dasein*, the appearing of economic objects in a specific mode of production – the capitalist institutional framework. In thinking about objects and their corresponding relations, we are thumping into a completely different onto-*logy* of appearing; one that is at work in Badiou's objective logic of appearing. Such a logic of appearing is, first of all, a mathematized logic grounded in a special branch of mathematics – category theory – and secondly, a localized logic of a concrete situation, world or institutional framework, presupposing inexistence of a totalized and universal logic. Let us introduce the initial thesis of the analogy: if Badiou postulates being as inconsistent multiplicity and multiplicities as being-there appear, embedded in relations among objects of a particular situation, then we shall connect the products of social labour in an interrelated structure of objects – a specific mode of social exchange.

Before we delve into the logical reconstruction of the value form, let us briefly overview Badiou's proposal for *materialist dialectic*. He draws a strict demarcation line, distinguishing the reigning paradigm of contemporary times depicted with the axiomatic statement: *There are only bodies and languages.* A duality that goes by the name of *democratic materialism* – a plurality of individual bodies, of a human animal endowed with language of universal juridical and normative equality.[12] To this duality Badiou supplements a *Third* term that marks the gap between the *Two* in the statement: *There are only bodies and languages, except that there are truths.* This will also be our guideline in the reconstruction of the value form. The part *except that there are truths* for Badiou implies thinking of a new objectivity, i.e. the analysis of objects and relations among them that "appears" as the truth, leaving us with the analogical question: How does the money form and objectivity come to appear in the gap of dialectical duality (content and form of commodities). Badiou introduces his "Greater Logic" in terms of objects and categories corresponding to mathematical category theory with which he conceptualizes the notions of transcendental, object, relation, etc. Each of these finds

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[11] Helmut Brentel, *Soziale Form und o¨konomisches Objekt: Studien zum Gegenstands- und Methodenversta¨ndnis der Kritik der politischen O¨konomie*, Opladen, Westdeutscher Verlag , 1989.

[12] Alain Badiou, *Logics* of *Worlds*: Being and Event II, trans. Alberto Toscano, London, Continuum, 2009, pp. 2.

its corresponding concept in *topoi*.[13]

We have already sketched our starting point and can now enforce it with Badiou's words: "In category theory, the initial data are particularly meager. We merely dispose of undifferentiated objects (in fact, simple letters deprived of any interiority) and of 'arrows' (or morphisms) 'going' from one object to another. Basically, the only material we have is oriented relations. A linkage (the arrow) has its source in one object and target in another."[14]

α) OBJECTIVE PROCESS

Are we not converging here to Marx's embryonic form of value, the Simple form: x commodity of A = y commodity of B? Everything is standing in its place: two objects (with a corresponding quantum) and their specific relation or "arrow" of equivalence.

The upper structure can be shown diagrammatically in the following way:

$$xA \longrightarrow yB$$

We will for the present moment omit the assignation of different quanta to commodities A and B, while presupposing the qualitative equivalence as one of many possible relations between two objects, where: $f: A \rightarrow B$; thus obtaining $f(A) = f(B)$. We have already pointed out that Marx discerns products and commodities, the former as use values in themselves, the latter as values in exchange. In exchange, different useful things take on a uniform shape, i.e. that of an object, becoming qualitatively equal objects of exchange – identical value-objectivity. Badiou, for instance speaks of the difference between things and objects: first being as pure inconsistent multiplicities without any qualitative predicaments, while the second are existent, being-there in a world, and behaving on different logical regimes of worlds measured by degrees of identity. The structure of two categorically identical objects (e.g. different in use value, identical in terms of value) and functional relationship (morphism) between them is

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[13] Badiou follows the basic definition of mathematical structure – *topoi* – as a local dimension of possible mathematical universes, i.e. a topos naturalized on set theory. Put alternately, it follows the *logic* of categories, where each universe subscribes to its own logic, definitions and schemas based on geometrical relations between objects. Accordingly, such a logic is definitional (and mathematics axiomatic), so by taking set theory, we engage in a well-pointed topos, where every difference is recognized in a point. In the case of a network of points and functions acting upon them, we speak of sheaves over topological space that forms a category – i.e. a topos. In the end, bringing a new axiomatic definition for set theory and creating a new categorial foundation of mathematics. *cf.* Robert Goldblatt, *Topoi: The Categorial Analysis of Logic.* New York, Dover, 2006.

[14] Alain Badiou. *Briefings on existence a short treatise on transitory ontology*, trans. and ed. Norman Madarasz, New York, State University of New York Press, 2006, pp. 145.

defined as *isomorphism*. A single object-identity is ascribed with an arrow originating and finishing in the same object, e.g. object A. Isomorphism makes two objects categorically indiscernible, in as much there exists an arrow between them. And it holds true that the reversed function returns null action. Now to our setting of the logic of value form: on the one hand, the products of labour are posited as things of different characteristics, shape, colour, composition, etc., and as commodities brought for market exchange in terms of "social" value on the other. Exchange in the capitalist mode of production knows only the latter type of qualitative determination of products of labour. Consequently, an isomorphism introduces a bijective[15] mapping, i.e. a reversible arrow f^{-1}, A ← B, having $f \circ f^{-1} = id(A)$. A value form example would be an act of two traders exchanging 20 yards of linen for 1 coat and reversing their deal, by exchanging the merchandise back. After the act they achieve their initial state of exchange. Although value does not yet acquire its own existence in actuality as something independent, it is nonetheless implicitly present in the value expression, which we shall express in the following commutative diagram:

$$A \xrightarrow{\quad f \quad} B$$
$$1(A)=1(B)\circ f \qquad 1(B)$$
$$V[1]$$

We denote the unity of use and exchange value, i.e. value as object V, and relate identity functions 1(A) and 1(B) to objects A and B. We have two objects of exchange put in a functional relation by subsumption to universal property of value. Indeed, things as objects possess a multiplicity of different sensuous characteristics that make them useful and readymade for exchange, but on the other hand, the identity functions depict the exclusive elementary belonging of the value-object (V) to a contingent object of exchange. We have said that the social form of labour assumes a two-fold definition, use value as a thing, (exchange) value as a (social) relation. Since one object (A) is exchanged for another object (B), it puts it into a relation. It therefore also acquires a value property. Such a universal property (V [1]) is ascribed to any commodity in the circulation process and it represents the unity of use and exchange value ("marking of an elementary unity in an object"). In category theory, it is called a *terminal object*. Such

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[15] The basic mapping functions include: injective: $\forall x_1, x_2: x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$, surjective: $\forall y \in B$, $\exists x \in xA: y=f(x)$ and bijective: being both injective and surjective and simultaneously invertible $f^{-1}: B \rightarrow A$.

an object has a unique (monomorphism) arrow leading to it from any object (a → 1). It defines a belonging of a universal element (value) to all objects exchanged in the circulation process. Before we continue our presentation of relating together objects and the universal equivalent, we must dwell on the dual character of social labour – value-creating labour [*Wertbildende Arbeit*], i.e. the unity of abstract and concrete labour.

Marx attributed a dual nature to commodity: the use and exchange value, from which it can be concluded that commodity-producing labour should also have a dual nature – concrete and abstract labour. Qualitatively different concrete labours end up in qualitatively different products of labour – use values –, which once put in circulation as commodities attain an (exchange) value. By means of abstraction, Marx unfolded value-creating labour into abstract labour (stripping all concrete labours to their universal character) – *a source of value*: "The coat is value only to the extent that it is the expression, in the form of a thing, of the human labour-power expended in its production and thus in so far as it is a jelly of abstract human labour – abstract labour, because abstraction is made from the definite useful concrete character of the labour contained in it, human labour. […] But as value this linen is a jelly of this same human labour."[16] This "jelly" of human labour is the *sine qua non* for the existence of form's value-expression, making the use value in equivalent form a form of appearance of *linen-value because linen relates itself to the material of the coat as to an immediate materialization of abstract human labour*. Such "expenditure" of human labour can be monitored in any product immediately after we perceive a value-substance emerging in exchange. It follows that all objects of exchange bear for-themselves the unity of concrete and abstract labour.

Let L denote those subobjects of exchanged objects that have the property of value creation, positing form-content to products brought up for exchange:

$$
\begin{array}{ccc}
 & T[1] & \\
{\scriptstyle x}\swarrow & & \searrow{\scriptstyle y} \\
L & \xhookrightarrow{\quad f \quad} & A
\end{array}
$$

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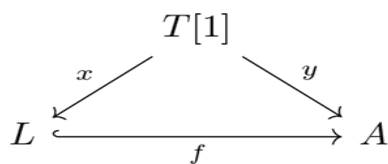
[16] Karl Marx, 'The Value-Form', p. 136-137.

We will say that if f presents a subobject (arrow from source L) with a (partial) injective mapping ↪, such that: y ∈ A, iff ∃x ∈ L; we get y = f ∘ x. This scheme represents the jelly of human labour, i.e. a set of individual concrete labours or combined parts of labours for commodity production. On the other hand, we have defined the diagram of *value substance*.

Until now we have examined the relation between commodity and labour, their dialectical unfolding from simple to expanded form, shown why an inverse mapping from many expanded forms cannot lead to a determinate universal equivalent (because we still have a multiplicity of simple and mutually exclusive forms, leaving A to randomly express its value in any of them), so we must now finally turn to the scheme of the universal equivalent. Marx considers the general form (form III) as a new form, differentiating it from the lesser developed two; this does not imply just reversing the position of two concurrent commodities, but much more, a shift in the form itself. Inverted places, where all of the commodities now express their value in the One (before it was one commodity expressing its value in many others) mean precisely the emergence of a general equivalent, *a concrete universal*. The former scission of labour (concrete and abstract labour) has another consequence: the emerging place of the universal equivalent is also the place of the general extension of labour [*Verallgemeinerung der Arbeit*]. If we condense the final result of the mutual exclusion process, we have A as any given commodity and B as a set of commodities such that B = {A$_2$, A$_3$, … A$_n$}, making it a whole other world of commodities with the exception of subtracting C, a general equivalent. This gives us the following diagram:

$$A \overset{f}{\hookrightarrow} B$$
$$v \searrow \quad \downarrow P$$
$$C$$

Object C acquires the property of a universal, general object, a moment that acknowledges its moment as a sensuous-supersensuous thing – a *concrete universal* – formally the relation f from one object A to many-counted-as-one object B followed by element (P)roperty, the value in the materialized form of general equilibrium that makes the triangle commute; v = P ∘ f. In other words, herein the shift in form occurs, logically manifested with the change in operators from *or* to *and*, i.e. from the disjunctive form of all accidental expanded forms to their conjunction. Once settled in, the general form fixates the concrete-universal object in equivalent form against

multiple other Ones in relative form. All of these *counted-as-one* in sum turn out all value relations for a particular world of commodities. Every particular commodity presents a part, or put differently, is accordingly an inclusion function ↪, a subobject of a commodity universe B. Now, enhanced with value property, our commodity A is ready for determinate quantitative relations to any other commodity, A_2, A_3, … A_n. Our last diagram represents the first half of value-generality's becoming; i.e. a construction of a *universal* object from the world of commodities. The second half, in effect, presupposes the transposition of the form itself, a mode as how to *think* the gateway from *n* expanded forms to *one* general form. To achieve this, we have to think the following in unity: the subobject of concrete labour jellied in the commodity, whilst at the same time forming a relation against the general equivalent. Every concrete labour has for-itself attached also the notion of abstract labour (as a generic part) and their unity accomplishes value substance. The content of this form must enter the *valuation* through equivalent form in order to be apprehended as part of the commodity exchange. If the opposite is the case, than we are confronted with a mere product, a use value valuable only to the producer himself. Since by definition all the relations are unique up to isomorphism, we shall use the *pullback* operation to get from disjunctive exclusion to reciprocal conjunction. The square L-A_1-M-$V[1]$ is called a *Cartesian square* in which a universal cone in L by pulling pairwise functions $V[1] \rightarrow M \leftarrow A_1$ is a pair $V[1] \leftarrow L \rightarrow A_1$ such that $T \circ 1(L) = v_1 \circ x$. If we now presuppose a Boolean codomain for every valuation in M, $[0,1]$ i.e. true or false, then we get a scheme where L is a subobject with elements of concrete and abstract labour, the chain A_1, A_2…A_n representing the objects of commodity exchange and M for now representing a universal equivalent (not yet money) and a valuation object, making $V[1]$ a terminal object (a definition of value). We get:
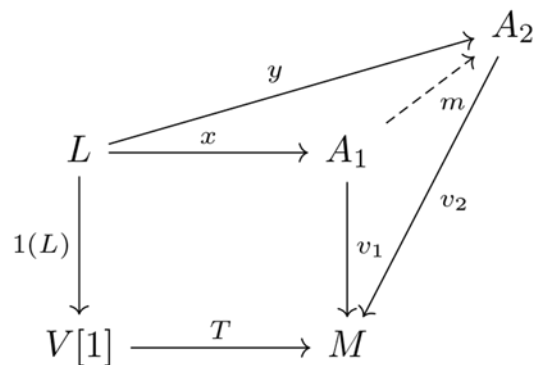
The scheme contains a formal representation of the exchange process in the

capitalist institutional framework. A set of different commodities is a general product of social human labour, spreading accordingly to higher levels of specialisation and expanding the world of commodities – A_{n+1}. Every single commodity understood as value is a jelly of human labour and they all share the expenditure of the commodity labour-force. On the other hand, this objectification of human labour presupposes a definite quantum of labour time invested in the production of a single commodity. If we say that the upper triangle L-A1-A2 commutes, this means that there exists a determinate relation between two commodities and objectified labour, i.e. the expenditure of labour-force in terms of the working day. There exists a qualitative and quantitative unity relation between the two, so that we have $y = m \circ x$, inducing mutual exchangeability, inasmuch as there exists content-wise some subobject L, whose elements (social and concrete labour time) are put in relation. Thus far we have shown that every object of exchange contains a subobject of labour as its part, or put in Hegelese, a reflection of essence posited as common ground. From now on, we continue in terms of value-objectivity: commodity is a bearer of dual nature – use and (exchange) value – and contains the presupposition of different commodities to observe value, inasmuch as they are an *expression of the same social substance, human labour*. The dawn of value's form of appearance. The reflection of labour and objectified value is posited as something equal using the commodity form, the juxtaposition of two commodities in direct (asymmetric) relation with a reflection in a third term. This "excluded" third is the equivalent commodity object, its natural form a common value image of the world, its bodily-shape a sensuous incarnation and a general social materialization of all human labour achievement. Countless equations that form the general form of value equate labour. For its own value-expression, the expenditure of human labour-force has to construct a new object (M), a universal equivalent, *de facto* connecting the nodes of relations and quantities between commodities for them to exist. In the preceding paragraphs, we have already set out the axiom of the source of value with the identity arrow 1(L), using monomorphism from $A \rightarrow 1$, a unique vector leading from object to its terminal, value V[1]. Recall that we initially decided on a Boolean-type logic, conceding the values for propositions to be either [T]rue or [F]alse. There is no third choice. We have also postulated that all objects (commodities) are "content-wise" the jelly or crystals of human labour. Each object (and for that matter, a thing) as far as social exchange goes, is *accountable*, if true [T] follows from the next conditional clause: *is the objectification of human labour-force expenditure in time*. All objects not submitting to this condition remain on the level of use values in itself, i.e. valuable only to their possessors. The opposite course of action comes to those who verify their social existence in the commodity-for-commodity exchange, becoming

(exchange) values transmitted among traders via a universal equivalent. Let T be a function pertaining to all subobjects: $V[I] \rightarrow M$, which outputs to object M the value {true} for all subobjects that meet the conditional evaluation. All remaining mappings not withstanding this evaluation are referred to as {false}. We are left with the triangle A_I-A_2-M, producing the form of object exchange and their valuations[17] against the universal equivalent. Indeed, since our mode of being is from the outset enveloped in a capitalist institutional framework, making the central object of our analysis the commodity exchange mediated with a universal equivalent, or as we shall later see, money. Our aim here is a retroactive three-fold "pull-back" of: commodity exchange (1) together with the constitution of value (2), the latter being mediated with labour (3). A pullback is said to be a limit for the diagram L-A_I-M-$V[I]$, a square that is also known as a *pullback square* or *Cartesian square*, a simultaneous operation of pulling back T along v_I and v_I along T to "produce" a relation between the subobject of commodity exchange L and the universal equivalent M. We postulate that L is the limit of our Cartesian square, i.e. the limit of our category $\mathcal{C}$. A limit in a category literally means a topological place from where all the objects of a category are universally "seen". In our universe, we have [L]abour as a universal subobject (being value-substance) of every object. Finally, we can conclude that there exists a functional *analogy* conjoining the "subobject classifier" or "truth value object" with the immanent logic of the value form:

$$
\begin{array}{ccc}
B & \xrightarrow{\ !\ } & 1 \\
{\scriptstyle f}\Big\uparrow\Big\downarrow & & \Big\downarrow{\scriptstyle T} \\
A & \xrightarrow{\ \varphi_B\ } & \Omega
\end{array}
$$

The characteristic function φ_B represents an indicator function for all those elements of subobject B that by verification also belong to object A and by rule assigns the value [T], otherwise value [F]. Pulling back along T and φ_B (outputting value [I]

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for all x ∈ B; B ⊆ A) leads to classifying elements in subobject B[18]. See how Marx derives our analogy:

> What was the way in which linen was metamorphosed into universal Equivalent, actually? By the fact that it displayed its value, first in one single commodity (form I), than in all other commodities in order in a *relative* way (form II), and thereby all other commodities reflexively displayed their values in it in a relative way (form III). The simple relative value-expression was the seed out of which the universal Equivalent-form of linen developed.[19]

Marx had formulated his method of inquiry in the following manner: deriving from empirical data, he *retroactively* extracted (pulled back) from commodity exchange mediated with money all the implicit moments, abstracting to simple or accidental form. All the previous forms (I, II, and III) are presupposed in form IV, the only "true and visible" form of the capitalist mode of production. Our aim is to delve into an analogous form, juxtaposing the structure of exchange with mathematical structures of categories – elementary relations, subobjects, and an object and a subobject classifier. All the above mathematical structures can form different elementary toposes. Some in particular are known as *Cartesian closed categories*[20], which have some specific properties for modelling random topological universes and draw crucial universal and particular consequences. The presented ability in analogously reconstructing such a universe based on elementary concepts, valid in *topoi*, supports the thesis on the strength of Marx's method of inquiry into social structure.

β) SUBJECTIVE PROCESS

There is one more parallel we have to draw regarding Marx's construction of the money form (form IV) and Badiou's usage of the *axiom of choice*[21]. The first step is to once more return to the simple or accidental form of value, one with a one-to-one correspondence (1:1) between two commodities. The next step is the expanded form, a one-to-many relation (1:N), whereas the commodity universe expands to (countable)

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[18] The constructions of a "limit" in a category are represented by a variety of cases: a terminal object, equalizer, Cartesian product, intersection, inverse image… For practical reasons, there is much use of the "pullback" or "fibered product".

[19] Karl Marx, 'The Commodity. Chapter One, Volume One, of the first edition of Capital', p. 33.

[20] A Cartesian closed topos is a category consisting of limits and co-limits for every finite diagram; there exist terminal and initial objects, products and co-products, pullbacks and pushouts; it has exponentiation and a subobject classifier.

[21] Formally denoted: Let S be a family of non-empty sets that has a choice function; $\forall S\ (x \in S, x \neq \varnothing) \Rightarrow \exists f,$
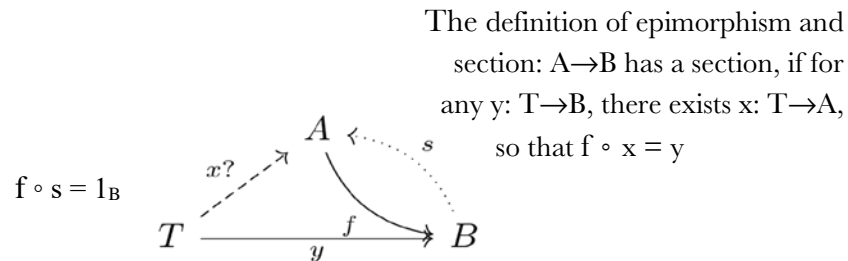$\forall x \in S, f(x) \in x.$

infinite limits. The gateway from expanded to general form is done with a qualitative turn from 1:N to N:1, entailing a new content-form [*Formgehalt*]. The whole world of commodities now has its value-body in the materialization of an excluded "thing", a commodity becoming a universal equivalent. The whole world of commodities is *counted-as-one*, out of which a commodity (a Universal Equivalent) is subtracted. We have seen above why this type of process should be characterized as an objective process; formally, it is a reciprocal and reiterative exclusion of individual commodities until the last one is left standing. But what actually happens on the pathway from general to money form? We could have easily concluded that the universal equivalent is already always money, merely a signifier substitute. Marx has something else in mind: he opts for a historicized interpretation of gold as a convenient commodity assuming the role of money. His concluding remark is also a (just) cause for different historical readings and interpretations regarding the birth of the money-object. On the contrary, the shift to money commodity should be seen as a pertinent operation for potential worlds of commodities. Undoubtedly, there is a multiplicity of contingent worlds in the capitalist institutional framework, as there are many different other institutional frameworks. In ours, on the other hand, the *choice* which commodity and in what form it takes the place of money is far from insignificant. We have already pointed to Marx's quote where he rather peculiarly says: "But only the action of society [*gesellschatliche Tat*] can turn a particular commodity into the universal equivalent."[22] The action of society always already presupposes that the money-commodity resides in the notion of the universal equivalent. In order to fully grasp the consequences of the "action of society", it should be understood as a *subjective process* of contingent choice. In other words, we are confronted with the *choice* of a subject. We are faced with the following two questions: What is at play in the transitional steps from the universal equivalent, excluded from the world of commodities, to money commodity? What operation leads us from excluded but individual existing materialization of value to the *internal exclusion* of the money commodity?

We will further elaborate this point and turn to the categorical analysis of mappings between sets. Arriving at the general form of value, we were presented with a many-to-one mapping. Many different commodities are valued on the predicate of embodied labour in a single commodity – the universal equivalent. The exclusion of this particular commodity is its structural necessity. But the final step from a universal equivalent (always a contingent commodity, depending on the comings and goings of social contracts) to a money commodity is the *ex nihilo* action of society on a *third* commodity, one always retraced back to the whole world of commodities. This kind of

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[22] Karl Marx, *Capital: Vol. I,* p. 180.

problem can be traced to the concept of *section* in category theory, or to put it somewhat differently, a special surjective mapping from many commodities to one and finding its right inverse – since money is also an element of the commodity universe – by relying on the axiom of choice. If we consider the mapping f: A→B, then we have to find a mapping corresponding to g: B→A, its right inverse, so that for some element it holds: f(g(b)) = b; b ∈ B. An inverse like this has a special trait – the existence of the subset $A_B \subseteq A$[23]. Elevating the discourse to category theory, there is an epimorphism which loosens the conditions for an inverse function, i.e. a section in the domain based on a choice function.

The definition of epimorphism and section: A→B has a section, if for any y: T→B, there exists x: T→A, so that f ∘ x = y

$f \circ s = 1_B$

Without progressing further into formalities and in relation to our problem, the existence of a right inverse conditions a section (function s and elements in subset A_B). Viewed from the world of commodities, it opens the path from a contingently excluded commodity operating as universal equivalent (world of commodities explicitly non-All) to an internal exclusion of the money commodity (world of commodities seen as totality, a Whole with an internal exclusion of money, implicitly non-All). Integrating the money commodity in the domain of the world of commodities is possible via the right inverse, implicitly admitting the axiom of choice. The question is how to choose the most appropriate commodity from all the potential commodities serving as money. The social action among a subset of *potentially* adequate commodities, i.e. the ones with suitable natural properties for a money-object, finally attributes to money commodity its final full functionality. The choice is needed while the universal equivalent is objectively the last standing commodity, but is nonetheless still only a general materialization of value. The money commodity is obtained recursively from the subset of potential commodities. Once the elementary choice inside the section is made, the externalized equivalent side of the value equation is obliterated and we end up with a self-sustained and unified commodity universe. The axiom of choice as an action of

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[23] Subset with property: ∀b ∈ B; there exists a non-empty set $A_B \subseteq A$, such that ∀a ∈ A_B, f(a) = b. In our case, it represents all of the necessary object-properties (permutations in the unity of concrete labours plus abstract labour) pertaining to the inception of a money-object.

society guarantees the existence of an inverse return from the set of universal equivalent {1} to section A_B in order to finally choose the representative of wealth. Consider the following mapping:
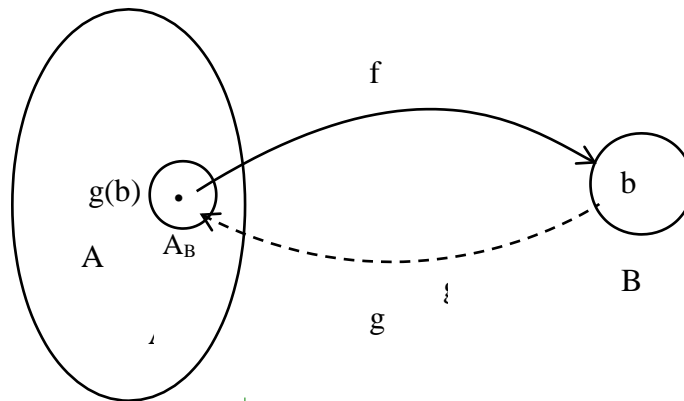
The recursive movement from universal equivalent object b to g(b) in the subset A_B needs to reassert all object-properties pertinent to money-object functionality to finally finish as a money commodity – f(g(b)) = b. To summarize: there exists no necessity in extracting a concrete money-commodity from the universal equivalent commodity, what is needed is *choice*. Formally, it comes down to an inverse operation utilizing the axiom of choice to find such an object. Marx speaks of "struggle for a monopoly position" or seeks refuge "in the action of society" regarding the introduction of money in exchange, while actually proposing a *subjective process*, a societal intervention into conducted exchange relations.

There is a strong parallel between Marx's constitution of the money-object based on the objective and subjective processes and Badiou's thinking of subjectivization[24]. What unifies these two discourses is the power of intervention provided by the axiom of choice. Badiou calls for subjectivization in the first instance as an intervention together with an operator of faithful connection – the name of the event as the result of an intervention and faithful connector to this new name-event in its generically truthful consequences. But didn't Marx already envisage this type of dialectic in his analysis of

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[24] Subjectivization as far as *L'Être et l'événement* (1988) is concerned, or the knot of subjectivization and subjective process in *Théorie du sujet* (1982).

value form? The objective process of distinguishing a universal equivalent is a nomination of a commodity-on-the-edge, the longevity of which is ascribed solely to the subjective process of fidelity to its maintaining key functionalities. The choice intervenes in this second part, both in Marx and in Badiou. Here is Badiou:

> The hypothesis I advance is the following: *within ontology, the axiom of choice formalizes the predicates of the intervention.* […] The undecidability of the event's belonging is a vanishing point that leaves a trace in the ontological Idea in which the intervention-[B]eing is inscribed: a trace which is precisely the unassignable or quasi-non-one character of the function of choice. In other words, the Axiom of Choice thinks the form of [B]eing of intervention devoid of any event. What it finds therein is marked by this void in the shape of the unconstructibility of the function. [25]

For Badiou, the axiom of choice links together the event, the subject, and an individual[26] and open the contingent gap for emerging subjectivization. What is at stake for Badiou, and for us following Marx and the value form, is seeking out the *conditions of possibility* for subjectivization. A completely contingent act performs the action of society in transforming a casual commodity into a money-commodity – with decisive consequences. In the case of Badiou: the inaugural naming of the event, followed by a forced expansion of a situation, a positing of an inextricable place and finally, a connection to a forthcoming subject of fidelity. In the case of Marx: the dawn of Capital, a presupposed random choice of a commodity with self-valorizing capabilities, and the sublation of money-form for capital-form. Value becomes an occult ability, it becomes a substance creative of value, because it is itself *de facto* value; money in the form of capital endorses value as an "automatic subject"[27]. Fidelity becomes a crucial moment in both approaches to the subject: with Badiou as a consequence of undecidability and tied to a place of intervention supplementing the immanence of a truth. On the other hand, with Marx, a theological-materialist instance of money without any forced intervention – the blissfulness of money already imposes its eternal piousness.

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[25] Alain Badiou, *Being and Event,* trans. Oliver Feltham, New York, Continuum , 2007, pp. 227.

[26] *Cf.* Ashton, Paul. & Bartlett, A. J. & Clemens, Justin, *The praxis of Alain Badiou,* Seddon, Vic, re.press, 2006, pp. 72.

[27] Our aim is not to seek parallels between Badiouian and Marxian definitions of a subject. The aim is to point out the immanence of a specific structural logic at work – implicitly with Marx, explicitly with Badiou.

LABOUR FORCE AND CAPITAL

What remains is the presentation of two special commodities: labour-force and capital. Labour force, i.e. labour capacity, is considered by Marx as a specific commodity, being both use and exchange value. Use value means living labour capacity to produce (surplus) value, giving its products exchange value (as objectified labour), one that can be attained by the owner of the means of production (the capitalist). The immanent value of labour capacity is, unlike other commodities that devour it as a result of social labour, precisely *sui generis* value. But in order to exist and reproduce itself, it must have its primordial origin – its means of subsistence is the person itself – so that it is capable of becoming objectified. It can be said that apart from sustaining and reproducing labour force nothing is more elementary in the relation production-distribution-exchange-consumption. Its existence and that of material wealth relies solely on the following: "As William Petty says, labour is the father of material wealth, the earth is its mother"[28] From the point of view of commodity exchange, the labour force contains no generic content, since it is the source of value. The process of commodity production therefore begins only after the consumption (alienation of labour) of labour force takes place. We will accordingly name the commodity labour force the *initial object* of the commodity universe. An initial object has the formidable property of maintaining relation to any other object in the universe, since it is the *void*, an empty set object. It follows: an initial object [o] is such that for any set A there is exactly one mapping o → A. We denote o as an initial object, corresponding to set theory {} or ∅ sign. The initial object is the dual or co-limit for a terminal object T [1]. The commodity labour force cannot withhold any *concrete form of labour*, itself being living labour, which would count as input to its existence. Conversely, all other commodities, including capital, presuppose inputs. Defined in such a way, the labour force object has a generic characteristic of inducing value, which as a corollary implies that it has no subobjects. Marx outlines this capacity of creating value to belong exclusively to productive expenditure of a possessor's living organism ("brains, muscles, nerves, hands, etc.") and concludes: "It is not labour which directly confronts the possessor of money on the commodity-market, but rather the worker. What the worker is selling is his labour-power. As soon as his labour actually begins, it has already ceased to belong to him; it can therefore no longer be sold by him. Labour is the substance, and the immanent measure of value, but it has no value itself."[29] Reflecting on set theory, we already touched upon an empty set, introducing the sole subobject of an object, which we we can now relate to the concept of abstract labour, or obversely: *use value of labour force*. We have seen above how "crystals of abstract labour" are treated as values,

[28] Karl Marx, *Capital: Vol. I,* p. 134.
[29] Karl Marx, *Capital: Vol. I,* p. 677.

confronting us now with a particular commodity whose content (unity of abstract and concrete labour) is void, but interrelate to a web of crystal connections spreading through the entire commodity universe. In other words, the initial object appears in all differential multitudes of commodities as an isomorphic subobject. Following by definition: every object isomorphic to an initial object is itself initial. If we now proceed to "classify" this relation between subobject and object we encounter a paradox – in the case of Marx, the *birth* of surplus value. By definition we have an initial object [0] as the sole subobject of object L (labour force commodity) stepping into the exchange verification against the money-object. If we further add that generic labour as an immanent moment of the commodity labour force has no value itself, we obtain:

$$
\begin{array}{ccc}
0 & \xrightarrow{\ x\ } & L \\
{\scriptstyle 1(0)}\big\downarrow & & \big\downarrow {\scriptstyle v} \\
V[1] & \xrightarrow[\ T\ ]{} & M
\end{array}
$$

A problem immediately arises: the relation $0 \to 1$. Since its valuation in a bivalent logic cannot return **True,** therefore by verifying $0 \to 1$ it returns the function $T \circ 1(0) = F \circ 1(0)$ and $v \circ x =$ **[F]alse**; $1 \to M$. We get $0 \cong 1$, valid only for degenerate categories and cannot hold for Cartesian Closed categories obeying classical logic, in practice meaning that labour for itself has no value, i.e. $0 \to 1$ is false. On previous occasions of objects we had the [T]rue, an *identity* arrow, always returning the value true. The arrow from 0 to 1, i.e. from initial to terminal object, on the other hand, must maintain the opposite, a *difference* mediated by the negation of the true, i.e. false. Badiou – in his notes in *Mathematics of the Transcendental* – puts it this way: "This is the occasion to give the only *demonstration* that I know of the following strong philosophical statement: *if the universe is not degenerate, that is to say if it contains real difference, then the true and the false cannot be, as actions or arrows, identical.* Or in short: the true is intrinsically different to the false, since the Universe is a multiple.

Thus, the existence of difference in general induces the true and the false. The multiple as a figure of being (here the multiple of objects) requires the fundamental

logical multiplicity (at the very least, duality): truth-values."[30]

In actual fact, this paradox is present both in classical and neoclassical economic theorizing. Postulating labour force, which obtains its value in the shape of wages, thereby "forces" its proper valuation in exchange. Marx calls it "an expression as *imaginary* as the *value* of the *earth*" or an irrational expression "value of labour", which on the surface substitutes the value of labour force and has its form of appearance in the wage. What counts for us is the following revelation: labour being the source of value by its very own substance enters a paradoxical relation with all other commodities in exchange. Furthermore, there is no "way" of pulling back the valuation of labour force, since it has only monomorphisms and an empty set for its subobjects. Advancing the dialectic of the value form and adding the particular commodity of labour force, Marx sought to demystify the erroneous logic pertinent to classical political economy. Doing so, he also anticipated the forthcoming aporias of neoclassical economics, showing in opposition to the former two, the superficial character and the intertwining of incommensurable concepts. Besides the phenomenal form of the value of labour and the wage, both paradigms had no trouble in incepting an additional two mystified forms of appearance of surplus value: a simpler one, namely profit, and the most concrete, mediated, fetishized, alienated form, the interest rate. Casually known as the most irrational formula of capital, the latter – interest – introduces a form of price devoid of content or put differently, a reduction of content to pure abstraction; capital relations, externalized [*veräusserlichte*] into interest-bearing capital. There is another aspect we have to put forward: money in-itself bears a moment of *temporality*, concretely manifested in the interest rate. If we step onto the surface of commodity exchange totality, to its forms of appearance, we can follow Jacques Rancière's concepts of externalizing a fetishized nature of capital:

1. *Relation* by which of course should be understood relation of production, in so far as it is these relations that underlie the whole process;
2. *Form,* in so far as the form is that by which the relation is manifested, by which it is represented in actuality [*Wirklichkeit*];
3. *Origin* and the *limit* of the process;
4. *Motion* or *development* of the forms;
5. *Result.*[31]

[30] Alain Badiou, *Mathematicsof the Transcendental,* trans. and eds. A.J. Bartlett and Alex Ling, London, Bloomsbury Academic, 2014, pp. 79.

The list is telling in that we have all the conditions in place for conjoining (in a pullback) two different functionalities of money capital – profit (π) of commercial capital (CC) and interest (i) of interest bearing capital (IC) – to their limit, i.e. their source of value in the form of appearance *wage-labour* (w). We have functions $\pi \to M \leftarrow$ i, both of them being an element (i.e. determination) of money as capital (M):

$$
\begin{array}{ccc}
& & IC \\
& & \downarrow i \\
CC & \xrightarrow{\ \pi\ } & M
\end{array}
\qquad\qquad
\begin{array}{ccc}
w & \xrightarrow{\ \pi'\ } & IC \\
\downarrow i' & & \downarrow i \\
CC & \xrightarrow{\ \pi\ } & M
\end{array}
$$

Comparing the upper scheme with Marx's schema of metamorphosis, starting from the basic setting C – M – C, we come to the disappearance of money C – C; changing a random commodity to labour force, we get the expanded form: M – C (L and mp) … P … (C+c) – (M+m), P is productive capital. The difference between M and M' (M+m) is an absent relation, deriving from the fetishistic character of capital self-valorization. We have formally ascribed it the operation of pulling back money capital along commercial and interest-bearing capital to the limit, wage-labour. Marx's take on this in the *Urtext*: "The exchange through which money becomes capital cannot be its exchange with commodities [in general] but can only be one with its conceptually determined opposite, the commodity which is itself a conceptually determined opposite of it – labour."[32] It is precisely wage labour that produces both versions of money capital (commercial and industrial capital, interest bearing capital…), the former with the eradication of the difference between paid and unpaid surplus labour, i.e. the function profit (π) in regular business conduct, the latter in the function of *yields i*, deriving from financial instruments – stocks, bonds, options, futures contracts, etc. – in terms of inequivalence and asymmetry. On the one hand, wage labour affects the creation of value in the end driving the current of money capital to self-valorization, its automaticity, and even more importantly, the structure of profits and interest *retroactively* affects the *conditions of existence* for wage labour. Marx encapsulates these determining

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[31] *Cf.* Jacques Rancière, 'The Concept of 'Critique' and the 'Critique of Political Economy', in Ali Rattansi (ed.), *Ideology, method and Marx: essays from economy and society*, . London, Routledge, 1989, pp. 155.

[32] Karl Marx, (1858) 'The Original Text of the Second and the Beginning of the Third Chapter of "A Contribution to the Critique of Political Economy" (the *Urtext*)' in: Karl Marx and Frederick Engels', *Collected Works. Vol. 29,* London, Lawrence and Wishart, 1968, pp. 505.

circumstances as follows:

> The alienation [*Veräusserung*] of labour-power and its real manifestation
> [*Äusserung*], i.e. the period of its existence as a use-value, do not coincide in time.
> But in those cases in which the formal alienation by sale of the use-value of a
> commodity is not simultaneous with its actual transfer to the buyer, the money of
> the buyer serves as means of payment. […] In all cases, therefore, the worker
> advances the use-value of his labour-power to the capitalist. He lets the buyer
> consume it before he receives payment of the price. Everywhere the worker
> allows credit to the capitalist. That this credit is no mere fiction is shown not only
> by the occasional loss of the wages the worker has already advanced, when a
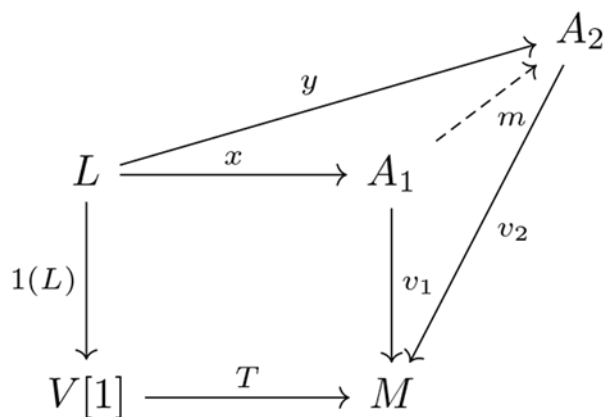> capitalist goes bankrupt, but also by a series of more long-lasting consequences.[33]

The consumption of labour force is simultaneously the process of commodity and
surplus production; we outline this "subtractive" character of relations in contemporary
economic science with the left diagram above. It depicts profits and interest as the
independent fruits of different capital variants. Marx sufficiently shows how the use
value of labour force, in-itself value-creating labour, remoulds into different forms of
capital, but does not give sufficiently strong emphasis to the recursive or retractive
process ranging from money capital to its source. From precisely this moment, wage
labourers not only encounter the abstract scheme of the capitalist's appropriation of
surplus labour, *a fortiori* they are confronted with a concrete form of their
bond/indebtedness to him[34]. Money capital appears as a mere thing, an object like any
other, where any relation to its self-valorization has vanished. During the sheer
development of this process (M + ΔM), mediated by the vanishing, the *irrational, a-
conceptual form* [*begriffslose Form*] emerges. Formally, this can be done by "collapsing"
functions or arrows defined as elements of particular objects – since both elements and
arrows can belong ($\in$) to an object of a category – reducing relational arrows to
elements. Not grasping this concealment of reduction leads to informational
deficiencies of value form. Contemporary economic theory simply rectifies this
manqué by jumping straight to price mechanism, in turn mapping the asymmetric
relation of labour and capital into new, higher aporias. *Material relations between persons
and social relations between things*.

If we return one last time to our classification schema of exchange:

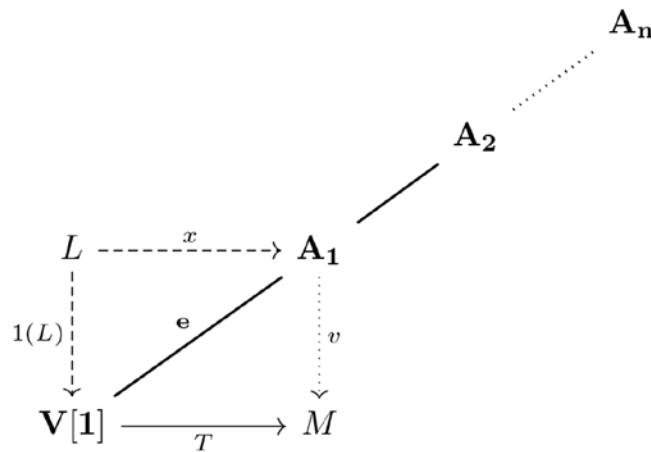
[33] Karl Marx, *Capital: Vol. I,* p. 277-278.

[34] In a contemporary capitalist institutional framework one only needs to think of different forms of debt:
from pension funds, savings schemes to ordinary bank deposits. All of the accounted stand for very
different schemes in terms of time periods, etc.

$$
\begin{array}{ccc}
L & \xrightarrow{\ x\ } & A_1 \\
{\scriptstyle 1(L)} \downarrow & & \downarrow {\scriptstyle v_1} \\
V[1] & \xrightarrow{\ T\ } & M
\end{array}
\qquad
\begin{array}{c}
A_2 \\
\nearrow^{y} \quad \nearrow^{m} \\
v_2
\end{array}
$$

What actually appears on the surface of the exchange process? While Marx addressed the problem of form with his *commodity fetishism* by stating: "It is only by being exchanged that the products of labour acquire a socially uniform objectivity as values, which is distinct from their sensuously varied objectivity as articles of utility"[35], the exchange process does not posit the above schema in appearance as a whole; it leaves some to surface, still others remain fetish-like concealed. People in the exchange overturn their labour products to mutual value-equality, i.e. based on isomorphic objects the various kinds products (first time equal as values) are equated as values or different kinds of labour as human labour – their common characteristic, labour in the abstract. So in the end, the superficial social relations between things add the mystification of social structure in that they conceal the topological underpinnings of production relations of men. With Rancière, we point out: "This absence of the cause is reflected by Marx as a mere distance. It is linked to the disappearance of mediations, obliviousness to the inner determinations of the process."[36] How can we formally *think* such a disappearance, obliviousness? Put alternately, how does one think the transformation of social productive processes into things with material properties?

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[35] Karl Marx, *Capital: Vol. I*, p. 166.

[36] Jacques Rancière, "The Concept of 'Critique' and the 'Critique of Political Economy'", p. 170.

$$
\begin{array}{ccc}
L & \xrightarrow{\ \ x\ \ } & \mathbf{A_1} \\
\downarrow \scriptstyle{1(L)} & \ \ \ \mathbf{e} & \downarrow \scriptstyle{v} \\
\mathbf{V[1]} & \xrightarrow[\ \ T\ \ ]{} & M
\end{array}
$$

If we *transpose*, i.e. map, the relations and objects over the diagonal of exchange (e), we get a torsion moment of two lacks being concealed[37]. Diagonally opposed and internally excluded, we get the subobject labour (L) on the one side and a commodity from the commodity universe, i.e. money, on the other. Their relational connections overlap, or, paraphrasing Marx, the money relation conceals one part of labour in particular, that which the wage labourer produces free of charge. The actuality of economic exchange realized in prices is represented on axis m, making its final result simply $A_1 - A_n$ or $C - C$, while the structural implications of both their sources and value mediation remain at a certain distance. "What is lost in fetishism is the structural implication that founds the distance of the thing from itself, a distance which is precisely the site at which the economics relations are in play."[38] The topological structure of this double lack converges on three discourses: Marx, Lacan, and Badiou.

We have at work two functionalities: the relations of production and monetary exchange (circulation); enveloped in the fetish of commodity form. The category introduced above corresponds to Marx's *flattened* general formula of productive circuit movement:

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[37] Here, we also refer to Alenka Zupančič and her parallels with the Freudian and Lacanian reading of the dialectic of desire.

In *Seksualno in ontologija (2011)*, Zupančič re-introduces Lacan's interpretation of Hamlet: the desire that troubles Hamlet is not his desire for Mother, but the other way around, Mother's desire (Mother as a desiring subject). It transpires that the subject's desire answers to the mother's enigmatic stance by offering its own lack. Precisely this starting point of the dialectic of desire ties the two sides of primordial repression – the objective and subjective side. The lack in the Other is covered by the subject's own lack (i.e. the lack of its own signifier), thereby activating the repression in a twofold manner: via the subject's associative chains and through drive filling the gaps in the constitution of reality. p. 143.

[38] *Ibid.* p. 163.

$$L$$
$$M - C \quad \ldots\ldots\ldots\ldots \quad P \quad \ldots\ldots\ldots \quad C' - M'$$
$$mp$$
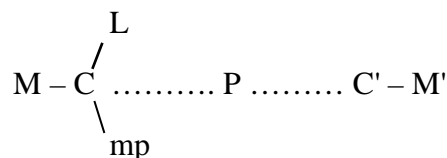
The formula represents conditions pertaining to a totality in general relation and linking together production, distribution, exchange, and consumption. The way in which all of them are put in the capitalist mode of intercourse is captured by Marx in the singular moment of their unity and manifested in circulation. Simultaneously, circulation dialecticizes the formal process with the power of contradicting one process against others in their totality. If we turn to our classification schema, we can disentangle the determinate terms of this relation: the upper left of our category introduces the production side, an injective inclusion of the subobject of labour to every object of our economic structure (triangle $L - V[\mathrm{I}] - A_\mathrm{I}$). Exchange (presupposing distribution) is a bivalent valuation of every object, whether it possesses the (exchange) value-property, identifying it as a commodity and exchanged for money and in the end satisfying a particular need (triangle $M - V[\mathrm{I}] - A_\mathrm{I}$). Axis e represents the unity of all moments in exchange, its surface in constituted reality, presiding over $A_\mathrm{I} \ldots A_\mathrm{n}$ or simply A (commodity universe) in the circulation process of the capitalist institutional framework – an objectivized *Dasein* of economic categories. We have tried to show all the moments of circulation in a unified structure, relating with arrows from production to distribution and further on exchange, in order to encapsulate the processes at their end – consumption – returning them to the initial point, running a ceaseless motion. This closed circular movement *eo ipso* exhibits the teleology of the capitalist mode of production. We have postulated for all of the relations in our economic structure to be isomorphic, i.e. retaining their universal properties (limit), being one time monomorphic (function x is injective) or second epimorphic (function v is surjective); the first preserving all differences among objects accordingly to the universal property (initial object $\varnothing$) of labour capacity L, the second preserving identities of object(s) A *vis-à-vis* M. We have a whole class of monomorphisms (variations of labouring capacities and activities), evaluating their products in relation to the money commodity. Besides, we also have an epimorphic transformation from a multiplicity of commodities in obtaining a particular one, acting as the universal equivalent and consequently as money, inscribing (indiscernibly and after a subjective choice) itself back into the commodity universe.

We have before us the economic structure in its entirety: the internalization of relations of production into value-objectivity (a thing), externalization of the money

exchange relation in the form of a characteristic function, evaluating all of the commodities against a single, money commodity. An eligibility verification of products endowed with universal property (operating on a bivalent (Boolean) logic) that enter monetary exchange, while maintaining the form of appearance in simple commodity exchange. In the category theory universe, such a mathematical structure is called a "subobject classifier", a "truth value object" or a Lawvere-Tierney object, whereas for Badiou, it represents a regime – *transcendental of a world*.

The most concrete and fetishized form of interest-bearing capital, the phenomenal form of interest, presents a special case of inversion and concealment of the represented economic structure. The relation between labour and interest-bearing capital presents itself as an immediate commodity relation of sale and purchase. It is perceived monotonously in every-day situations. But if we go on and recursively disclose its apparent a-conceptuality, we can posit that labour L is overdetermined with money as capital. The use value of labour together with the exchange value of money pulled-back thus form the first condition of capital. It goes along as the most cunning relation: in the most developed form of interest bearing their encounter accounts for a *determinate time dimension*. A subtractive process is running on the flip side of this time dynamic: the redistribution of surplus value (overturned to profit), realized in the forms of entrepreneurial profit or better still, financial yields – overflowing the asymmetric relation of value. Why asymmetric? *The current composition of equity and debt relations relying on yields (i) constitute the future structure of relations between labour and capital, which will retroactively disclose the initial composition of these concrete relations only in the unforeseen future.*

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