

ON THE RELATIONSHIP BETWEEN INFORMATION AND HEAT

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ABSTRACT: Where information is measured by the total surface area of a system of objects, it thus has a physical representation or manifestation. Since all heating occurs at the boundary of a collection of objects, heat must be related to information in a very specific way so as to allow for phenomenon such as melting, cooling, and the thermal expansion of objects. This essay argues that information (entropy) is nothing less than proportional to the rate at which something is heated with respect to time, and attempts to draw meaningful implications therein.

KEYWORDS: Information; Heat; Entropy; Science

In *Matter-Information Equivalence*, we were able to identify that the amount of information a mass is composed of is related to its surface area. If the mathematics failed to convince you of this fact, consider the following analogy. You are putting together a jigsaw puzzle piece by piece, when you realize that the last piece is missing. Frustrated that all your hard work will have gone to naught, you scatter the puzzle pieces across the table. After doing so you notice the following fact. When the pieces were together in a regular pattern of rows and columns, each piece shared a boundary with another piece, and the outer perimeter was minimal. But when you scattered the pieces, they no longer shared boundaries, and thus the total perimeter of the system increased along with the *entropy* or “disorder” of the the system. Thus, because entropy increased, it would take more information to describe the positions of the pieces as well as their orientation in space. More entropy, more disorder, more perimeter. But as you solve the puzzle, it takes less information to describe the pieces, because they are in a regular, predictable pattern. Less entropy, less disorder, less information, and less perimeter.

In three dimensions, perimeter becomes *surface area*. Take a large chunk of ice

and smash it into a hundred tinier chunks. The collection of tinier chunks has more surface area than the larger chunk, and thus takes more information to describe.

But now, let us switch gears so to speak and discuss *heat* and make explicit its relationship with entropy in both the informational and physics-related sense of the word. Heat flows from an object which is hotter to an object or environment which is colder. A large chunk of ice whilst sitting under the sun will melt into a liquid. In other words, the entropy of the universe (system + environment), or systemic unpredictability, always increases. This phenomenon is commonly known as the *Second Law of Thermodynamics*. On a molecular level, when heat flows from hot to cold, the hotter molecules, which have more kinetic energy, bump into the colder molecules, transferring to them kinetic energy. The entire system agrees on an equilibrium temperature, but all the while increasing the disorder within.

We will consider two simple systems. Consider first a large ice cube like one that a classy bar might put in a warm glass of whiskey. For this closed system, the environment (the whiskey) is warmer than the system (the icecube) and thus heat moves from the whiskey to the icecube, which will eventually melt the ice cube. Now consider the same amount of ice in mass and volume, but broken up into tiny chunks, placed in a glass of whiskey at the same temperature. The question is, which system will melt faster or *at a faster rate?*

Assuming that the temperature difference between the ice and the whiskey in both cups is the same, our intuition inclines us to believe that the second whiskey cup, wherein the ice is broken up, will melt faster. This is in fact the case (and exactly why bars use large ice cubes!), but why? The answer cannot be “because the second cup has less matter.” since we established that the same amount of ice was used in both systems. Since the temperature difference is the same between the ice in both systems and the whiskey, this also cannot be the reason. Furthermore, we must entertain the hypothesis that the only remaining difference between the two systems, their difference in *surface area*, may be the reason why the larger ice cube melted more slowly than the collection of smaller ice cubes.

Ah, but as we now know, surface area is a measure of the information contained in a system. Indeed, just as we saw with the jigsaw puzzle, a completed

puzzle, with all its regularities, is easier to describe than a scattering of the jigsaw pieces. In the same way, a large cube of ice contains less information, and thus has lower entropy, than a collection of smaller chunks of ice, which exhibit a higher degree of entropy. Since the smaller chunks contain more information, and since they melt the fastest, we arrive at an interesting conjecture. The more information, or entropy, a system possesses, the faster the exchange of heat from the environment to the system. This can be expressed as follows:

$$S = k * dQ/dt \quad (1) \quad \text{where } S = \text{entropy and } Q = \text{enthalpy in joules}$$

As of now, this law is only conjecture, because it has not been experimentally tested in any sort of rigorous way. What it essentially tells us is that the current entropy of a system (such as big ice cube or a collection of smaller ones) is a change in the amount of energy of the system with respect to time. It also succinctly expresses the fact that as surface area (information) increases, the rate of heating increases. Not to oversimplify this law, but this finally explains one of the biggest urban mysteries known to man: why bigger ice cream cones melt faster than smaller ice cream cones! A bigger ice cream cone has more surface area than a smaller scoop, and thus the rate of heating is initially much larger than the smaller scoop, and thus dribbles down the side of the cone and makes a mess (surely we have all witnessed this). Of course, it must be noted that the entropy of a system compared to its environment is *not constant for many real systems*, just as a large ice cube gets smaller and loses surface area over time. If you decrease the surface area, you don't change how hot it is per se, but rather the rate at which its gains or loses heat, all else equal.

Not to be too philosophical, but we can attempt to draw a significant conclusion regarding the nature of the *bit* or binary digit from equation 1. Since matter is information, information in the context of matter, that is, a physical context, now has empirical meaning. Because the surface area represents a measure of entropy, information must in coincide be a measure of the rate of heating or cooling of the object in question. If a system requires a large amount of information to describe it, it will heat or cool at a faster rate than a system with comparably lower entropy. Thus, there is a sense in which a bit can be identified

with a change in the energy of a system over time. A thing with more bits is more susceptible to energy changes than a system composed of less information.

Practically speaking, we can easily measure the change in entropy of a system using only a measurement of temperature change. If we take a coffee cup and place a thermometer inside of it, the change in temperature over time will be in proportion to the entropy of the coffee. If we put a tight lid on another cup of coffee, we reduce the the surface area of the coffee, and we can safely say that it will take longer to reach the room temperature than the first cup, and thus accurately reflects the reduction in entropy or surface area. This is likewise what happens when we heat a piston; the surface area of the cylinder (and volume) increases a fixed amount as the gas particles do work on the piston in accordance with rate at which the boundary is heated. If the source of heat is removed from the piston, dQ/dt decreases, entropy decreases. Notice this does *not* in any way break the Second Law of Thermodynamics, which say the total entropy of the universe must be increasing. An entropy decrease of the system (the gas) is accompanied by an entropy increase in the environment. In other words, the environment, not the gas, becomes warmer upon compression.

The definition of a *change in entropy* as described in most physics textbooks is as follows:

$$dS = \text{integral}(dQ/T) \tag{3}$$

Thus a change with respect to time yields

$$dS/dt = \text{integral}(dQ/dt)/T = \text{integral}(S/kT)$$

Moving “dt” over to the right hand side, we get

$$dS = (1/kT)* \text{integral}(S dt)$$

$$T dS = (1/k) * \text{integral}(S dt) \tag{4}$$

Why does equation 4 make sense? On the left, we have the temperature of the system times the change entropy. Through unit analysis, we find that the left hand side leaves us with energy (joules). By a similar unit analysis, the constant k must have units of

$K = s/k$ (seconds/kelvin). The integral on the right has units of $J*s/k$. Thus, multiplying the integral by $1/k$ we find that we are left with joules also on the right. In fact, where the left hand side represents the *work* or *energy* of the system,

the right hand side, being the integral of entropy, corresponds to the *volume* of the system. Thus, just as surface area can be identified with entropy, so too can volume with work. Furthermore, the definition of a change in entropy as described by our current knowledge of physics nicely supports our conjecture in equation 1.

Consider now a metal sphere that we wish to heat for a given amount of time. Since surface area is a measure of entropy, we have that:

$$S(t) = 4 * \pi * R(t)^2 = k * dQ/dt$$

Thus,

$$\longrightarrow dQ = 4 * (\pi / k) * R(t)^2 dt$$

Integrating, we find that

$$\longrightarrow Q = \text{Integral} (4 * \pi/k * R(t)^2 dt) = 4/3 * (\pi/k) * R(t)^3 = (1/k) * V(t) \quad (5)$$

Equation 5 tells us that the amount of heat transferred to a system at a given time is directly proportional to its volume $V(t)$ as a function of time during the heating. As you heat the metal sphere over a given time, it is seen to expand. This indicates that the amount of heat transferred to the sphere is directly proportional to its volume; *more volume, more energy added*. The rate of its expansion must therefore be proportional to its surface area, and increases over time in this case. Thus, as the sphere expands, it becomes easier and easier to raise its temperature. Now, if we hold ice to a flame for a given amount of time, we notice a similar effect, except the ice cube will get smaller and smaller as it is heated. Of course, this is commonly attributed to the ice cube changing phases, but if the ice cube alone is our system, we would say as its volume decreases, so too does the heat added to the system. Another way to view this is that as the surface area of the ice decreases, the heat added during one time interval will be greater than the heat added during a later time interval.

Even more precisely, we can use *differential equations* to accurately describe such scenarios. Since the surface area changes along with the volume, the temperature must also change in this manor. The following is a common formula from chemistry:

$$Q = mc \, dT = mc (T_f - T_i) \quad (6) \text{ m is mass, c is specific heat}$$

For the metal sphere,

$$Q = (1/k) * V(t) = 1/k * 4/3 * \pi * r(t)^3$$

$$Q' = dQ/dt = (1/k) S(t) = 1/k * 4 * \pi * r(t)^2$$

Dividing equations gives us the differential equation

$$Q / Q' = r(t) / 3$$

$$\longrightarrow Q - (r(t)/3) Q' = 0 \quad (7)$$

Thus, if we know how the radius changes with time, we can derive the enthalpy and the change in enthalpy with respect to time, or entropy. Knowing Q , we can also know how hot the thing is by knowing its specific heat and mass.

While the deepest secrets of the bit have yet to be uncovered, hopefully this essay shows that information has a meaning that is rooted in a physical manifestation. On the one hand, entropy can be regarded as systemic unpredictability, which can be *represented* as the surface area of the object or system in question. The greater the surface area, the more unpredictable a given system is. By virtue of a system's high entropy or greater surface area, its temperature can increase or decrease quicker than that of a system with a smaller degree of entropy. Thus, there is a reason to believe that the amount of information an object contains is its *willingness or ability to change temperature*. Objects with a lot of information will change temperature easily whilst heated, while objects with less information will resist changes in enthalpy.