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IS SPACE DISCRETE? AN INQUIRY INTO THE REALITY OF PLANCK LENGTH AND ITS PHILOSOPHICAL IMPLICATIONS

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ABSTRACT: In this paper, I examine the philosophical assumptions subtending the newly emerging concept of the smallest possible chunk of discrete space — Planck length, employing the method of historical analysis, hermeneutics, and phenomenological investigation.With the metaphysical presuppositions undergirding Planck length revealed, I attempt a discussion of the philosophical implications of the discrete space.

KEYWORDS: Planck Length; Discrete Space, Loop Quantum Gravity; Infinity; Husserl

Space has always been considered infinitely divisible and thus continuous throughout the western philosophical traditions beginning with the ancient Greeks. Take for example: it is one of the most fundamental *a priori* assumptions in Euclid's *Elements* that space can be infinitely extended or diminished continuously. Meanwhile, the ancient thinkers in the east do not differ much from their western counterparts in terms of the perception of space either. A fellow philosopher and good friend of Zhuangzi named Huishi[1] depicted in TianXia of the Miscellaneous Chapters in *Zhuangzi*[2] speaks of the infinite divisibility of a one-chi stick[3]. However, this notion of space has also been questioned and

[1] 惠施

[2] 庄子,杂篇天下

[3] 一尺之棰，日取其半，万世不竭 （A one-chi stick, half of which is taken away every day, will not be exhausted in ten thousand generations). Although here Huishi is actually concerned about the infinite divisibility of a material thing (hyle/matter) rather than space, it seems that such a feature of material things is indeed predicated upon the infinite divisibility of space. Chi is a Chinese unit of measuring length. It is

challenged since the very beginning. Zeno of Elea, a disciple of the pre-Socratic philosopher Parmenides who denies the reality of motion, was the first and foremost (at least that we know of) to disturb our notion of space with his famous paradoxes. Although Zeno did not question whether space could be divided infinitely, he maintained that the infinite division of space was able to be actualized[4]. To defend the never-ending and non-actualizable infinite divisibility of space, Aristotle launched various refutations against Zeno's claims throughout his *Physics* and *Nicomachean Ethics*. However, the apparent absurdity of Zeno's paradox, despite inciting some deeper inquiries into the profound complexity of space, does not convince us that space is composed of infinite individual quantifiable points. In fact, challenges, especially those like Zeno's paradoxes, often contribute more to better developed and articulated explanations on why space is continuous instead. Since the idea of a discrete space seems to be so counterintuitive, and, moreover, its implication that motion is merely an illusion seems to be so ludicrous and absurd to anyone with even the minimal capacity of sense perception, discrete space appears to be more of a threat to be resolved rather than a legitimate alternative to the continuous space.

Therefore, although there is no shortage of "challenges" of this sort throughout history, space was still widely received as being continuous until the rise of Loop Quantum Gravity (LQG) theories[5]. Attempting for the grand unification of general relativity and quantum mechanics, LQG, instead of building upon the foundations of either of the two theories, decides to return to and reconstitute the most primary ground of any study of physical science -- the ontological status of space -- to genuinely challenge its fundamental continuity. The most radical presupposition that subtends the field of LQG is that not only space is real and discrete, but also that the elementary constitutive unit of space[6]

often translated as the Chinese foot. The length of one chi varied significantly throughout Chinese history, ranging from 20 cm to 36 cm.

[4] All of Zeno's paradoxes including Achilles and tortoise, flying arrow, dichotomy, etc. are predicated upon the ontological presupposition that space is composed of infinitely many indivisible points.

[5] Quantum gravity theories include a group of theories often classified under either string theory or loop quantum gravity. Though space at the Planck scale is crucial to both theories, only certain theories under loop quantum gravity propose the possibility of an alternative discrete theory of space. Therefore, I will take this model of LQG as the main subject of investigation in this paper.

[6] We would not draw too much of a distinction among the uses of space either as one-dimensional length, two-dimensional area, or three-dimensional volume. Since Planck length is at the heart of any type of space on the Planck scale, we will use it to denote space in general.

(the length of the smallest possible "chunk" of space) is one Planck length, which is roughly equal to 1.6 x 10^{-35} m, or about 10^{-20} times the size of a proton. But, before we delve into the investigation of the meaning and implications of Planck length, let us take a closer look at some groundbreaking moments in history of mathematics and natural science to see how they have revolutionized our conception of space, but, at the same time, how they have also left the continuity of space intact.

The first of such moments is the monumental invention of calculus by Newton and Leibniz (working independently from each other) in the mid 17th century. One may argue that the method of taking derivative and integral in calculus is actually a practice of actualizing the infinite; therefore it implicitly assumes the fundamental discontinuity of space without explicitly spelling it out; for calculus demands space to be cut into infinitesimal[7] "chunks" first and then to be added back together for us to have a "limit". However, I would not deem that such operation of calculus denies the continuity of space at its core. Firstly, calculus does not assume that there is a smallest discrete unit of space: although it seems to utilize such smallest unit implicitly as the infinitesimal quantity, it is precisely because the quantity actualized is infinitesimal, i.e. vanishing, that calculus can avoid being involved in the problem of assuming space as discrete. Moreover, under the framework of Newtonian mechanics, the background in which all calculations take place is the one and only absolute continuous space[8]. Hence,

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[7] The ontological status of the "infinitesimal" had been a long-lasting dispute among philosophers and mathematicians. Ever since the notion was first introduced by Leibniz, it had been under numerous vehement attacks, among which the most famous one was launched by Bishop Berkeley. It was only until the 1960s mathematician Abraham Robins could provide a supposedly rigorous foundation for it in his non-standard analysis. Thus, for as long as almost three centuries, students of mathematics could only go through the lengthy and complicated alternative method of limit (initially proposed by Newton as the nascent/evanescent quantities and subsequently developed by Bernard Bolzano, Augustin-Louis Cauchy, and Karl Weierstrass). But, for the purpose of this paper, I will simply use the terms "infinitesimal" and "limit" interchangeably without entangling myself with the details of the debate throughout the history of calculus.

[8] Much is to be said about the difference between the Newtonian calculus and the Leibnizian calculus. While Newton advocates the absolute space and time (throughout his magnum opus Principia Mathematica), Leibniz, on the other hand, thinks that space and time are relational instead (the Leibniz-Clarke correspondence). But, since Newtonian mechanics remained the generally acknowledged paradigm of modern physics until Einstein's theory of relativity, we, pace Leibniz, will simply take absolute space as

even if calculus requires space to be divided into "seemingly" discrete slices or disks, the absolute space as the continuous underlying background is left intact.

Nearly two and half centuries later, we welcomed a way more revolutionary and counter-intuitive theory of space. According to Einstein's theory of relativity, space (along with time[9]) is no longer a fixed background in which movable objects pass through. Instead, it is contracted with the objects in motion. The faster a frame of reference moves, the more contracted the space becomes. However, the contraction of space according to Lorentz transformation in special relativity still operates under the premise that space is a continuum; similarly, the curving of spacetime in general relativity does not require space to be divided up into individual discrete units to work either. Therefore, Einstein's theory of relativity, although forever changed our conception of space, time, and motion, still leaves the continuity of space untouched.

Thus, what inspired LQG theory to break away from the traditional perception of space and renounce its long-established continuity? To answer this question, we have to go back to the rise of quantum mechanics at the turn from the 19th century to the 20th century. Max Planck, the acclaimed originator of quantum mechanics and also after whom the supposedly smallest length is named, championed for a set of "natural" units in his presentation to the Prussian Academy of Sciences in 1899. Nevertheless, this is not the public debut of natural units in general. More than two decades ago in his lecture delivered at the British Association in 1874, Irish physicist George Johnstone Stoney already proposed a series of natural units utilizing only four base units, including the speed of light in vacuum, the gravitational constant, the Coulomb constant, and the elementary charge. Planck units only differ from Stoney units slightly: instead of using the elementary charge, Planck units include the reduced Planck constant h-bar and add the Boltzmann constant to the family. Although there are other possible systems of natural units, Planck units are the most well-accepted natural system in contemporary physics. It is as the length unit of this natural units system that

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the background prior to Einstein. But, in spite of the radical difference between absolute space of Newton and relational space of Leibniz and Einstein, they agree upon the continuity of space in general.

[9] I think it is not so far-reaching to accredit relativity as the first theory that establishes the intimate bond between space and time. By exalting the constant speed of light in the vacuum as one of the unquestionable assumptions of his theory, Einstein made space and time truly dependent on each other for the very first time.

Planck length is given to us.

But, let us pause for a moment before we begin to explore the empirical and ontological significance of Planck length directly. We must put into question the motivation behind Planck's (and Stoney's) persistent advocacy of a natural-units system. Since Planck length is one of the core members of Planck units, a close examination of the "superiority" of Planck units and natural units system in general will significantly aid our attempt to understand Planck length better philosophically. There are several questions that need to be addressed for our inquiry: why do we even need natural units? Don't we have enough units? Just think about the plethora of units which already complicate our daily lives: mile, kilometer, yard, Celsius, Fahrenheit…, so what is the point of creating the so-called natural units?

For the physicists working on quantum mechanics, the traditional units systems, as popular as they are, present a rather serious problem: they are not "natural". That is to say, these units are not given to us by nature, instead they were arbitrarily constructed by humans. Thus, as a consequence of the artificial construction, they are completely dependent upon prototypes that were randomly selected by humans as well. Prototypes are artificially made objects that are used as standards of measurements. We see that in most cultures the earliest prototypes are often human body parts[10]. For example, a foot, as its name indicates, is defined originally by the length of a foot; a Chinese chi, similarly, is firstly derived by measuring a human hand from the tip of the thumb to the tip of the forefinger. Not surprisingly, such a way of taking measurement creates confusions: after all, people do not have their feet and hands in the same sizes. Thus, we move to the next stage of standardizing units of measurement — finding objects that are most likely not subjected to change and making them prototypes. The most well-known example of such prototypes might be the platinum-iridium cylinder in Paris which still defines the standard kilogram for us even today.

As we can already see from the first two stages, the general tendency of seeking new systems of units is undoubtedly driven by our desire for the most universal and self-sufficient "language" in physics. From the rather arbitrary and

[10] In fact, these early prototypes are often created for a political-social-economical reason, i.e., to divide up the farmland, to trade in the market, and etc. Science is never pure in itself; it is well entangled with the world from the beginning, which we will address more thoroughly in the later section of this paper.

varied prototypes such as human body parts to the more rigid and unified prototypes such as metal rods, we are always on the path searching for a system of units that can reduce the constraints imposed by prototypes as much as possible. Thus, a system of units completely free from prototypes seems to be the ultimate goal in the field of physics, even in the domain of natural science in general.

How the natural units systems rid themselves of prototypes is through normalizing the so-called universal physical constants. Here we will only be concerned with the most predominant natural units system — the Planck units. As it is addressed briefly above, there are five fundamental universal constants that the whole Planck system is founded upon. Each of the constants is considered fixed; that is to say, they are not subjected to change under any circumstance. Take for example the speed of light in vacuum c: it is one of the only two assumptions that undergird Einstein's whole theory of special relativity. Surely, these constants are derived from empirical facts, but they are quickly being elevated and placed on the pedestal as the ultimate universal foundation for the entire realm of physics. Then, by normalizing these constants, i.e., making them equal to the dimensionless 1 (the unit), the equations in theoretical physics can be reconstructed and thus simplified. More importantly, since these building blocks in physics — these natural units — are not dependent upon prototypes, consequently the edifice of the field of study will be free from the fetters of "human caprice".

It might be the case when Planck proposed these units, he had in mind of a "universal language" given to us by nature. This endeavor of Planck is not dissimilar to an ongoing movement in philosophy to search for a universal language for logic and metaphysics initiated by Gottfried Wilhelm Leibniz. What is more, we may say that Planck's project even takes a step further from Leibniz's *Characteristica Universalis*. While Leibniz and his followers such as Frege are still concerned with perfect knowledge representation to all human beings regardless of one's native language or culture, Planck imagines a language that even transcends human perspectives -- for the physicist, this is the language of Nature[11]. As the supposed most universal language, the significance of Planck's system must

11 Hence, it is imagined by Planck as the only language to communicate with animals and even aliens.

surpass the mere boundaries of physics and present a series of questions in need of philosophical investigations.

Residing at the heart of the Planck units, we find the base of the system — Planck length. It is defined by three fundamental constants: the speed of light in a vacuum, the Planck constant (the reduced Planck constant), and the gravitational constant. The Planck Length ℓp is defined as $\sqrt{\hbar G/c^3}$: the square root of reduced Planck constant times gravitational constant over the speed of light in a vacuum cubed. Since all three constants employed here have fixed values, the Planck length is also fixed as a natural consequence, approximately equal to 1.616229(38)$\times 10^{-35}$ m. The derivation of Planck length is a rather interesting and unconventional procedure. Opposed to the traditional process -- pick a prototype which is as universal and rigid as possible and then set it as the standard measure, Planck simply plays with the five fundamental constants to see if any combination of all or some of them will result in a "chunk" of length, i.e., a result with "meter"[12] as its unit. No surprise, it turns out that there is only one way to combine these constants and the result of the equation gives us the Planck length.

Then, if the Planck length is neither empirically derived nor is it verified by experiments, why is it believed by many contemporary scientists and philosophers to be the smallest length possible? Undoubtedly, there are indeed physical manifestations which suggest the reality of Planck length as the smallest possible space, among which the most well-acknowledged one pertains to Heisenberg's famous uncertainty principle. At the scale of Planck length, it is meaningless to try to distinguish two points (positions) apart based on the calculation according to the uncertainty principle. Moreover, on the empirical level, if we attempt to investigate any distance smaller than one Planck length with physical experiments, i.e., sending a photon to the space to be studied, a black hole would form due to the high energy/mass (of the aforementioned photon) in comparison to the limitedness of the space we try to confine it in. Thus, no information can ever be revealed if we attempt to investigate any shorter

[12] "Meter" here only serves as a unit indicator of length, not that "meter" should be deemed more fundamental than Planck length as units.

distance than a Planck length[13].

Nevertheless, despite these physical manifestations championing for the legitimacy of Planck length, it is the rise of LQG theory that really places Planck length in the centerstage, making it a possible ground for a unification theory potentially. Thus, let us take a quick trip to the early days of the theory to see how Planck length achieved its paramount significance. In his *Three Roads to Quantum Gravity*[14], Lee Smolin offers a brief account of the founding story and early development of LQG. Smolin's narration provides us with a glimpse of how the Planck scale units have gained their theoretical and even physical importance. Attempting to unite general relativity with quantum mechanics, theoretical physicists strive to explain gravity at the quantum level without a recourse to any fixed background (i.e., a fixed space continuum is that on which quantum mechanics is built upon). Inspired by theorists working on QCD (quantum chromodynamics), Smolin and his colleagues, instead of working on theories that are dependent on a spacetime continuum, found their work on the assumption that space should be conceived as discrete. More radical than physicists such as Kenneth Wilson, leader of QCD, who conceive a discrete but fixed lattice of space, Smolin and contributors to LQG eliminate the dependency on background once for all, and moreover, they define space *as the interrelations among a set of discrete elementary objects*. That is to say: according to LQG theory, space is actually made of small chunks that are no longer divisible and each chunk of space is created by a set of relations of the most elementary objects (which are defined by them as the "loops"). No surprise, based on the calculations made by these physicists, each discrete piece of indivisible space is at the Planck scale. Thus, the smallest length is about the Planck length, the smallest surface is around the Planck area (the square of the Planck length), and the same extends to the three-dimensional space, Planck volume, as well.

If the "proofs" of the reality of Planck length by Uncertainty Principle and black hole formation are *reductio ad absurdum* — assuming a smaller space and demonstrating its absurdity, Smolin and his peers are attempting a much greater task — they no longer try to just prove the reality of Planck length, rather they

[13] We will address the philosophical implications of these positions in more details later in this paper.
14 The two chapters that are most pertinent to our discussion here on the discreteness of spacetime and Planck length are Chapters 9 & 10 "How to count space" and "Knots, links and kinks".

"elevate" Planck length to the status of the grounding principle of their entire theory. However, just as all roads lead to Rome, it seems that different approaches all lead to Planck length. Does this mean that the verdict has been reached on the reality of Planck length and we should celebrate the triumph of a discrete space?

Before we joyfully jump to the next "historical revolution" on the conception of space, we need to question deeper if there is any hidden assumption which necessarily leads to the "validity" of Planck length. First of all, for certain there appears to be the unanimous convergence to Planck length from seemingly different lines of work — uncertainty principle, black hole formation, and LQG, but instead of taking it as an irrefutable proof of the discrete space, shouldn't we entertain the possibility that perhaps all these theories inherit the same presuppositions as their starting point? As a unification theory, LQG cannot and does not wish to overturn every underlying principle in either general relativity or quantum mechanics. With integrating the two theories as its goal, LQG necessarily continues its work within certain framework of quantum mechanics. Since uncertainty principle serves as one of the defining principles in the entire field of quantum mechanics, it is not surprising that LQG, which still heavily relies on the framework and even "tradition" of quantum mechanics, should arrive at a similar position in terms of Planck length.

Another problem we should be concerned about pertains to theoretical physicists' implicit practice pursuing beauty and elegance when forming their theories. Aesthetic value, especially the simplicity of equations, is surreptitiously guiding physicists' work from the very beginning. The incentive of devising a simple elegant equation which can be put on a T-shirt, with the perfect example being Einstein's $E = mc^2$, is so strong that they are willing to introduce multiple extra dimensions to achieve an apparent simple formula.

Although Smolin only sparsely and elusively touches on the reason why he believes that space should be discrete and relative, we can still infer from his cursory accounts that a principle that has been implicitly ruling the field of natural science as the heuristic guide is at work here — Occam's razor or *lex parsimoniae* (law of parsimony). Despite its various formulations, Occam's razor could be simply stated as that among competing hypotheses, the simplest should always be selected. Smolin explicitly "announces" this hidden principle very briefly when he reflects on the discrete lattice proposed by Kenneth Wilson:

> If physics is much simpler to describe under the assumption that space is discrete rather than continuous, is not this fact itself a strong argument for space being discrete? If so, then might space look, on some very small scale, something like Wilson's lattice? (Smolin, 2001, 116)

As we can see, the main reasoning behind the shift from continuous space to discrete space might well be the law of parsimony. Since the "math" is much simpler under the presupposition that space is discrete, then there is no reason to adhere to continuity. The tendency to fall back on the law of parsimony as the heuristic guide exposes exactly the unquestioned and foregrounded prejudice operating behind the curtain in natural science. It is so rarely reflected upon that we almost forget that the law is not exactly a scientific law such as Newton's laws of motion or the second law of thermodynamics, rather it has its "humble" origin in aesthetics and theology. Although William Occam himself did not invent the principle, the "razor" is associated with him due to the effectiveness and frequency he used it. It is not far-reaching to include Occam's personal background as a theologian and the medieval tradition of conceiving God's work as the most simple and elegant into consideration, when we take a closer look at the law of parsimony. The medieval scholastic thought passed on to the early modern philosophers and natural philosophers, the geniuses of the 17th century as Whitehead puts it, such as Galileo, Descartes, Newton, Leibniz, etc., with the ultimate goal to imitate God's work on a smaller scale in mind, formed their work in accordance. For the scientists today, the principle of simplicity may strike as a scientific principle just as any other ordinary law, since the history and tradition it carries with itself is concealed and — in Husserl's words in *The Crisis* — sedimented. But we must find problems with the seemingly "innocent" use of it and question further the philosophical and metaphysical implications brought to the fore by it.

Finally, I would like to return to a discussion of the natural system, especially the faith placed on such a system. As we have mentioned earlier, the Planck system is deemed as free from human caprice, the ultimate universal language of nature. The hidden assumption at work here is a metaphysical position which privileges objectivity over any subjective experience. Even we have marched so far in quantum mechanics to admit the observer effect, which basically states that we cannot simply observe a phenomenon without necessarily changing it, somehow at a more fundamental level, we still believe that the most perfect science is the one without human "contamination" — a science free from presuppositions and interpretations. From Leibniz to Plank, the goal of the *summa scientia* is to let nature reveal itself with the highest clearness and distinctness so that everything is apodictic and nothing ambiguous is left for us to dispute over. With such weight placed on the Planck system and consequently on Planck length, we naturally want to believe the reality of it. We wish that there is something more to it rather than the result of playing around with some physical constants. Thus, when we stumble upon the physical manifestations, we are so eager to embrace the "reality" of Planck length and forget to question all the hidden assumptions for us even to get there in the first place.

But, let us bracket the reality of Planck length and discrete space for now and entertain the possibility that the space is indeed made of many discrete real "disks" that are called Planck length. What would be some obvious consequences if we take this position?

At the beginning of this paper, we introduced Zeno's infamous paradoxes that have been bothering philosophers on the nature of space for more than two thousand years. Now let us conduct a brief review of the historical responses to the paradox and then re-examine them with the newly acquired "knowledge" of Planck length.

Besides the terse refutation Aristotle provides in *Physics*, he provides another more in-depth critique of the paradox in Nicomachean Ethics. A detailed analysis of Aristotle's account was given by Robin Collingwood in his 1945 book *The Idea of Nature*. Collingwood remarks that the key to Aristotle's solution of Zeno's paradox lies in his conception of the non-homogeneous nature of motion, which builds upon the non-homogeneity of time and space. According to Collingwood, Aristotle considers it not a contradiction for "movements of a certain determinate kind" to be made of "movements not of that kind". That is to say, the whole movement of a flying arrow is potentially composed of an infinite number of instants of the arrow at rest and each instant does not qualify as actualized motion. Since motion is the ratio between space and time, space, similarly for Aristotle, can be potentially divided into an infinite number of indivisible mathematical points which have no parts, but each indivisible point cannot be considered as actual space.

Aristotle's refutation of Zeno relies on the very division of space into two distinct ontological categories: potentiality and actuality. But since the inauguration of the early modern tradition by Descartes which establishes the essence of physical space to be mathematical, the distinction between potential and actual space is annihilated. However, with the newly acquired hypothesis of Planck length, Zeno's paradox can be resolved without recourse to Aristotle's non-homogeneous space. It is precisely because Zeno's indivisible points are infinitely many, the paradoxes are able to stand as being paradoxical. In fact, the "indivisible point" employed by Zeno is a very subtle and complex philosophical and mathematical concept. In a certain sense, it is not dissimilar to Newton's "evanescent quantities" proposed in his *Principia Mathematica* or the "infinitesimals" advocated by Leibniz. The point itself, if we can borrow from Euclid's definition in the *Elements*, has no parts[15]. However, just as the Newtonian evanescent quantities or Leibnizian infinitesimals, it is real in the sense that Achilles or the tortoise must traverse. If we follow Newton's formulation, it is a quantity that vanishes. Or according to Leibniz, it is a quantity that infinitely approaches zero, or that is less than any given magnitude. Nevertheless, it is not nothing or unreal: rather it holds itself distinct from a total nullity. However, with space being conceived as composed of discrete chunks of Planck length, the ontological complexity of space brought out by Zeno's paradoxes is being taken away completely. Space no longer entails the infinite in it: we will no longer have to think of the universe as infinitely vast or being limitless; instead, it is just finite multiples of the Planck volume no matter how big the number of multiples is. Moreover, Achilles and the tortoise will not be trapped in an impasse either: since they can only "hop over" multiples of Planck length each step instead of traversing every "point" of the continuous space, for certain Achilles can easily take over the tortoise with a greater velocity.

But, is Planck length truly a "real" being? How should one understand its reality if it is indeed "real"? Since this paper is not concerned with the experimental verification but rather with the ontological status of Planck length,

[15] The first definition of Book I of Euclid's Elements. While the term "part" (μέρος) can be interpreted in various ways, we will stay with the reading that a point, distinct from a line or a figure, is something that has no magnitude.

we will explore the "reality" of the so-called smallest space from a philosophical perspective. The question concerning the "being" of Planck length and space in general is indeed a philosophical question, or if we may, a metaphysical one. We will conduct the discussion on the implications of Planck length from the side of the traditional metaphysical framework, i.e., from the perspectives of epistemology and ontology respectively. Even though we are not especially concerned ***the*** metaphysical question: the "τὸ τί ἐστι" (the "what is") question raised on the ground that presupposes a fundamental split between experience of an object and the essence of it , a close examination on the epistemological and ontological implications of a discrete space still offers valuable insights into our study of space.

As we have discussed earlier in this paper, the general opinions that tend to accept Planck length as being real usually fall into two categories: one from the perspective of experimental measurability and the other from the theoretical calculability. These two positions serve a perfect entryway for us to examine Planck length from both an epistemological and an ontological standpoint respectively.

For people who speak from the first position, they are convinced of the reality of Planck length because of the impracticability of an epistemological knowledge of anything smaller than the aforementioned quantity. Let us refresh our memory of their argument: imagine a thought experiment to actually measure a space smaller than the Planck length, we need to shoot a subatomic particle to the space. Since the energy/mass of the particle would be too great for a space smaller than the Planck length, a black hole would be formed and thus, nothing will be revealed to us. This position actually confuses the boundary between epistemology and ontology. According to this line of reasoning, it may be true that we will never know anything about any space that is smaller than Planck length, but does this validate their ontological claim that Planck length is surely the shortest length? It seems that Planck length, from this perspective, simply resembles the ultimate Kantian noumenon or thing-in-itself. However, even in the perfect Kantian world, the thing-in-itself only delimits our epistemological knowledge. We will not deny that Planck length might be at the upper demarcation of our epistemological capacity as human observers, but can we jump so rashly to the final judgement about its ontological status as the smallest constitutive element of space?

On the other hand, scientists speaking from the position of the second category champions the veracity of Planck length employing Heisenberg's uncertainty principle, especially the mathematical formulation of the principle. Because the wave-particle duality of elementary objects such as electrons, we cannot know its precise position and exact momentum simultaneously. Hence, with one property (either position or momentum) being fixed, we can only estimate the other with a probability function. But, according to the probability function, the positions could no longer be considered as being different from each other within one Planck length. Therefore, scientists in this category defend the reality of Planck length as the smallest chunk of space. Unlike the previous one, this argument seems to be an attempt at the ontological level. However, it is an ontological judgement that solely relies on the metaphysical priority of physical laws and their mathematical formulation. Principles and laws (in this case Heisenberg's uncertainty principle) are exalted to the highest ontological status along the metaphysical ladder. Consequently, the mathematical formulations of such principles also achieve the ontological status as *ens realissimum* (the most real being). Nevertheless, the metaphysical presuppositions implicitly at work in the field of mathematics are completely neglected and ignored if we place our faith solely in mathematics. We ascribe reality to Planck length because the probability function says so, but what are the unquestioned assumptions of the probability function? Was it only first proposed as a mathematical approximation for the phenomenon? How about the troublesome idea of "infinity" that is at play at the core of modern mathematics since the very invention of calculus by Newton and Leibniz?

As we can see from above analysis, either from the position of epistemology or ontology within the traditional metaphysical framework, the "reality" of Planck length is fundamentally problematic. It is certainly dubious to simply accept Planck length based on the above reasons. But, let us once again entertain the possibility that Planck length is indeed real, and see where the speculation can lead us in the metaphysical realm. One direct consequence (which we have already hinted on in our analysis concerning Zeno's paradoxes) would be that it seems there will not be any "infinity" in the universe. If this is the case, does it mean that we have been toiling the infertile soil over the idea of the infinite for two thousand years? Does the rich philosophical concept giving rise to so many philosophical and scientific theories is merely a phantasm of our illusion?

These questions seem to put us in an *aporia*, nevertheless, since we do not confine our inquiry within the narrow delineation of traditional metaphysics, we can turn to Husserl and phenomenology for a different approach or attitude. So far, we have only been looking at the Planck length and its either continuous or discrete nature of space through the lens of modern physics, however, we have not questioned the metaphysical commitments with which modern science, especially mathematics and physics, operates. Thus, phenomenology will help us see the risk to place our investigation solely upon a ground of mathematized natural science.

According to Husserl, natural science beginning from the Age of Enlightenment does not really reveal to us the true nature of space except for its abstract attributes which can be exploited by certain mathematical utilization, i.e., that which can be subjugated under the rules of calculation. In the section on Galileo (§9) of his unfinished posthumous book *The Crisis of European Sciences and Transcendental Phenomenology*, Husserl is particularly concerned with this covering-up of the life-world with mathematics:

> Mathematics and mathematical science, as a garb of ideas, or the garb of symbols of the symbolic mathematical theories, encompasses everything which, for scientists and the educated generally, represents the life-world, dresses it up as "objectively actual and true" nature. It is through the garb of ideas that we take for true being what is actually a method...It is because of the disguise of ideas that the true meaning of the method, the formulae, the "theories," remained unintelligible and, in the naive formation of the method, was never understood (The Crisis, 51-52).

We would certainly fall into Husserl's criticism of the scientists and the educated if we indeed take for granted the nature of space only as it is "disclosed" by mathematics and mathematical physics. Husserl does not deny that mathematical science discovers and discloses certain aspect of the world — the measurability and the calculability that are extremely useful for our practical needs, but he is more worried about what must be concealed as a necessary consequence for such discovery and disclosure: we dress up the life-world with mathematics, and take the garb of ideas as the real world. Thus, regardless of the illuminating insights on Planck length we gathered from the above analysis, we simply should not take them as the final judgement on the nature of space. Whether it is due to the black hole formation theory, or the natural consequence of uncertainty principle, or the "proof" from LQG, the evidence championing for the reality of the shortest possible length is without exception all derived from mathematics, and hence according to Husserl should only be considered as one method (not the only method) to look at the world and not the world itself.

Furthermore, according to Husserl, we would not be deprived of the idea of infinity even if there is no real infinity in the universe. In the third meditation of his *Cartesian Meditations*, Husserl proposed a new way to conceive the infinite. Instead of searching the infinite in the physical world or in mathematics, or in religion, e.g. the infinite universe, the infinite line, and the infinite being — God, we experience the infinite in our everyday life: our possibility of experiencing a worldly object is infinite. There are infinitely many ways to experience something; the possibility of experience it is always open. There is no guarantee that we would experience even the daily object tomorrow the same way we encountered them today. However, the openness is, on the other hand, not unbounded: there is a concrete horizon that delineates our possibilities of experiencing a certain object, but this horizon yet remains infinitely open, just like an asymptote that is well defined but never closes itself off. Therefore, if we take Husserl's critique and suggestion seriously, Planck length, or discrete space in general, as one concrete way to experience space, i.e., the way through the lens of mathematics and physics, should neither be dismissed nor worshiped.

In conclusion, we conducted an analysis of Planck length and the nature of space with an aim to expose the hidden metaphysical assumptions and presuppositions undergirding our conception of space in the frameworks of different theories in contemporary physics. Along the way, we realized a more extensive critique of the metaphysical prejudice subtending mathematical physics and the Enlightenment science in general must be performed. Thus, we turned to the work of Husserl, especially his *The Crisis of European Science*, and carried out a phenomenological critique of the current practice in science that places absolute faith and value in mathematics, i.e., attributing the most primary ontological status to formulas, equations, geometry, calculus, etc. After all, the value of questioning into the reality of Planck length in the philosophical attitude lies more in the bringing forth of the unexamined metaphysical commitments of contemporary physics rather than declaring a final judgment on the nature of space within the un-reflected scientific practices.

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