# RETHINKING THE METAPHYSICAL IMPLICATIONS OF QUANTUM DISCONTINUITY: THE ERROR IN OUR APPLICATION OF THE PRINCIPLE OF NONCONTRADICTION

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ABSTRACT: This essay argues that we struggle to understand the ontic meaning of quantum phenomena because we fail to realize that the discovery of quantum discontinuity, that is to say, the spatiotemporal randomness of quantum interaction, has brought into question our *application* of the principle of noncontradiction as a fundamental ontological law. It makes the novel proposal that the *real* (i.e., ontic) meaning of quantum discontinuity, the fundamental ontic configuration of our world and the problem of how the principle of noncontradiction should be applied in our world all equate to essentially the same question. The reason we fail to make this connection is because we conflate the application of the principle of noncontradiction as a fundamental ontological law with the idea of noncontradiction as an a priori truism. Once we come to terms with this basic error, we can then realize that the relationship of spatiotemporal continuity–discontinuity that defines the limit of measurable phenomena can also be understood to represent the *appearance* of the *real* fundamental configuration of our world. The simplest explanation for this *real* configuration is the emergence of causality from randomness. Bearing in mind, this fundamental ontic configuration would have to serve not only as the starting-point for our world, but also as the necessary connection and starting-point *for any knowledge claims within and about our world*, effectively including the origin of contradiction itself, the role classically played by the a priori truism of noncontradiction.

KEYWORDS: Ontology; Noncontradiction; Discontinuity; Quantum Theory

## SECTION ONE: INTRODUCTION

There is a distinction to be drawn between how we describe the phenomena we can measure and the way we describe the world beyond what can be measured.

Before the development of modern science, the latter was the subject of metaphysics and specifically ontology as the philosophy that concerned itself with the nature of Being and existence. It is science, after all, that brought us to the limit of measurable phenomena and made it possible to discover the *spatiotemporal randomness*—that is to say, the inherent *discontinuity*—of quantum interaction. Of course, philosophy continues to play an important role in our scientific study of the fundamental nature of the world, although its primary role today rests not so much with describing the ontic configuration of our world, which has become almost the exclusive domain of physics, but its epistemology and logic, and specifically the question of whether or how we can cognitively know the real world beyond the phenomena. The famous debate between Einstein and Bohr, for example, boiled down to this dilemma, and whether it is actually possible, even with mathematics, to access the fundamental workings of the world. 'Einstein is closer to Hegel […] in giving the primary role to the formation of concepts […] in this pursuit of a mathematically idealized representation of physical reality,' Arkady Plotnitsky explains.[1] Aligning himself with Kant, on the other hand, 'Bohr's epistemology is not based on any of these assumptions, in particular on the assumption that a description of or even conception of quantum objects and their independent behavior is possible'.[2] Understandably, perhaps, scientific orthodoxy mostly rejects Bohr's position as too nebulous and unsatisfying, preferring instead, like Einstein, to rest its hopes on mathematics. It is understandable, as well, especially given the unquestionable success of mathematics in being able to describe and predict the measurable phenomena —quantum mechanics, quantum electrodynamics, quantum field theory—that mathematics should inevitably be able to unravel, too, the enigma itself of quantum discontinuity and the apparent complexities of what lies beyond.

This essay argues that the solution to this problem could actually be simpler than we think, and that it may, in the end, just boil down to a rethink of the way the principle of noncontradiction is *applied* in our world as a fundamental ontological law. It is a novel proposal of this essay that the *real* (i.e., ontic) meaning of quantum discontinuity, the ontic configuration of the world and the question of how the principle of noncontradiction should be applied in our world all

1 Arkady Plotnitsky, *Reality without Realism: Matter, Thought, and Technology in Quantum Physics*. Cham, Switzerland, Springer, 2021, p.10.
2 Arkady Plotnitsky, *Niels Bohr and Complementarity: an Introduction*, New York, Springer, 2013, p.10.

essentially equate to the same problem. It is argued, as well, that the discovery of quantum discontinuity has brought to light a metaphysical error that has effectively hindered Western thinking since Aristotle, that is to say, the conflation of the *application* of the principle of noncontradiction to our world as a fundamental ontological law with the idea of noncontradiction as an a priori truism. Once we come to terms with this error, the simplest explanation for the contrary relationship at the limit of measurable phenomena between quantum discontinuity and the continuous causal structure of the physical world is the emergence of causality from randomness.[3] Not only would such a dynamic effectively precede our application of noncontradiction, but this *real* relationship could be expected to *appear* at the limit of observable phenomena as spatiotemporal continuity–discontinuity, that is to say, from within and as part of the same world. It is asserted that the metaphysical implications of this possibility represent the real import of this discovery.

In this essay, quantum discontinuity refers specifically to the randomness in space and time of quantum events, to be exact, the recorded traces of those events. An under-appreciated though defining aspect of both quantum physics and the argument in this essay is that quantum interaction only ever occurs in our observations as spatiotemporally random (i.e., discontinuous) events. *All* the properties that contribute to our understanding of quantum objects ultimately derive from the relationship between this measurable discontinuity (i.e., spatiotemporal randomness) and the continuous causal structure of the physical world, represented in our scientific observations by the experimental apparatus used to measure such events. The idea of particle–wave duality, for example, comes about entirely as a result of the effects of this relationship on the experiments designed to observe the behavior of quantum objects. The seminal example of this is the double-slit experiment. In such experiments, the resulting wave-like interference patterns and particle-like 'machine-gun' effects that give rise to the notion of quantum duality are composed exclusively of the accumulated traces of quantum discontinuity, in other words, literally thousands of such spatiotemporally random quantum events. The extrapolation of these

3 Arkady Plotnitsky, *The Principles of Quantum Theory, from Planck's Quanta to the Higgs Boson the Nature of Quantum Reality and the Spirit of Copenhagen*, Switzerland, Springer International Publishing, 2016, p.x.

patterns to the notion of particle–wave duality is then predicated on the assumption of a causal ontology, the existence of which, in turn, is almost universally taken for granted and presupposed by the application of the truism of noncontradiction to the world. It is the apparent certainty of this truism and the presumption, ultimately, of its natural application to our world that underpins virtually all efforts to interpret the relationship at the limit of measurable phenomena between quantum discontinuity and the continuous causal structure of the physical world.

To be clear, it is not the truism of noncontradiction itself that is in question, that the same thing cannot ontically exist and not exist simultaneously, or, as Aristotle originally formulated it, '[i]t is impossible that the same thing can at the same time both belong and not belong to the same object and in the same respect'.[4] Rather, it is the way in which this principle applies in our world and specifically, in the first instance, when applied as a fundamental ontological law. In this regard, it must be reinforced that the principle of noncontradiction, when applied in our world, is, *before all else*, a fundamental ontological law. 'Since Łukasiewicz [1910], this ontological version of the principle has been recognized as distinct from, and for Aristotle arguably prior to, the logical formulation […] and the psychological formulation'.[5] 'If the principle of contradiction were not an ontological law,' the early twentieth century German ontologist Nicolai Hartmann pointed out, 'and did not have validity in relation to the being of essences, it would be a violation of the essences by a tyrannical kind of thinking'.[6] The definition of 'essence' in this context, following Hartmann, 'can be summarised in the thesis that "what is" is essence' (OLF 76). It is asserted that, at its most basic, the initial connection and starting-point for any knowledge claims about our world is *not* the a priori truism of noncontradiction, but the necessary application of this principle as a fundamental ontological law, that is, as a fundamental law of Being.

It is the application of the principle of noncontradiction as a fundamental

4 Laurence R. Horn, 'Contradiction', *The Stanford Encyclopedia of Philosophy*, Edward N. Zalta (ed.), Winter 2018.

5 Horn, 'Contradiction'.

6 Nicolai Hartmann, *Ontology: Laying the Foundations* {1935}, Trans. Keith R. Peterson, Berlin, Boston, De Gruyter, 2021, p.76. Multiple citations of this book may be textually rendered as (OLF).

ontological law that provides the starting-point and ontic connection for Knowledge in our world, and thus also, by implication, the first law of logic *within our world*. Further, it is argued that this application is defined, almost certainly, by our world's ontic configuration rather than simply the a priori truism of noncontradiction. 'It is ontologically important from the outset,' Hartmann (OLF 252) explained, 'to understand the category of the "world" as *the encompassing category that it is*' (emphasis added). The concept of 'our world' is used in this essay to express this *all-encompassing nature of the world we live in*. This is similar, also, to how Heidegger defined the concept of 'world' in *Being and Time* (1927) to designate the 'world' "*wherein*" [*we*] "live"".[7] By definition, it is impossible for there to be any knowledge *in our world* other than knowledge *about our world*. For obvious reasons, it is considered 'logical heresy' to question the principle of noncontradiction as an a priori truism; indeed, as Aristotle originally formulated it, the principle of noncontradiction is an axiomatic law of logic. *However, in order for that principle and such logic, actually, to be applicable to our world, it must first be applied in our world, and that can only be done from within and as part of this same world*. In asserting the truism of noncontradiction as the a priori starting-point for knowledge about our world, Aristotle effectively conflated the idea of this truism with its application as a fundamental law of Being. If this is indeed the case, then such a realization carries profound implications for our metaphysical understanding of the world. And, it is an error that has only really become apparent with the discovery of quantum discontinuity.

## SECTION TWO: QUANTUM DISCONTINUITY IS A CRITICAL ONTOLOGICAL PROBLEM

It is because of the uncertainty that the discovery of quantum discontinuity has created with regard to the application of noncontradiction to our world, and thus to the initial connection and starting-point itself for any knowledge claims about our world, that, in the first instance, the correct approach to the ontic meaning of this discovery is ontological, not mathematical, logical, epistemological, or even phenomenological. It is argued that this problem constitutes, actually, the

7 Martin Heidegger, *Being and Time* {1927}, Trans. John Macquarrie & Edward Robinson, Oxford, Blackwell, 2001, p.93.

first metaphysical question, that is, the nature of the connection and starting-point for any knowledge claims about our world. This question is classically presupposed to have been answered by Aristotle with the a priori truism of noncontradiction.[8] It is the surety of this truism and the supposed certainty of its application beyond the phenomena that classically serves as the starting-point for all our knowledge claims about the world and presumably guarantees the initial connection of those knowledge claims to our world. Even non-classical thinking that questions this classical role of the principle of noncontradiction—for example, quantum logic[9]—still invariably presupposes the validity of this principle in our world as a necessary, indemonstrable truism and as a fundamental, a priori, law of logic and knowledge. Both classical and non-classical thinking alike invariably take for granted that the logic itself of the truism of noncontradiction guarantees its a priori application in knowledge claims about our world. This presupposition has become so ingrain into our thinking, as an axiomatic law, that we almost universally fail to realize that it is not actually this principle that initially connects knowledge to our world but the application of this principle, and specifically as a fundamental ontological law. It is this application that has become questionable because of the discovery of quantum discontinuity at the limit of measurable phenomena. Because a priori methods of analyses presuppose the logical validity in-itself of the truism of noncontradiction, when they attempt to analyse the ontic configuration of our world, they effectively locate themselves already within the, so to speak, system (i.e., our world) they are attempting to understand. One result of this is that they tend to confuse this ontic configuration with the effects that it has on the phenomena. This is why, in spite even of the accuracy of these methods in being able to analyse the phenomena as such, the ontic meaning of quantum discontinuity and the initial connection of all knowledge to our world is first an ontological problem and specifically an ontology that actively does not presuppose the initial application of noncontradiction to our world as merely an a priori truism.

Because it is this very first metaphysical question regarding the starting-point and ontic connection for knowledge to our world that is at issue, the most

8 c.f., Paula Gottlieb, 'Aristotle on Non-contradiction', *The Stanford Encyclopedia of Philosophy*, Edward N. Zalta (ed.), Spring 2019. Multiple citations of this article may be textually rendered as (ANC)

9 C. de Ronde, G. Domenech, and H. Freytes. 'Quantum Logic', *The Internet Encyclopedia of Philosophy*, June 4, 2023. This provides a good overview of the historical development of quantum logic.

appropriate method for analysing the ontological implications of quantum discontinuity is a critical ontology such as Nicolai Hartmann's that does not take for granted the initial application of the a priori truism of noncontradiction.[10] 'It is the foundational part of philosophy,' Hartmann (OLF 3) explained with regard to the subject matter of his critical ontology. 'I would have preferred the name "*philosophia prima*" (first philosophy) coined by Aristotle if there were any prospect of making it part of current vernacular again.' Of course, this age-old presupposition regarding the truism of noncontradiction played a significant part in shifting the primary contention of modern metaphysics from ontology to epistemology, first with Descartes and then notably with Kant's transcendental research program, and specifically the question of whether or how we can cognitively access the fundamental ontic configuration of reality. By the time Max Planck discovered quantum discontinuity at the beginning of the twentieth century this shift had already become a fait accompli. Even Husserl and Heidegger's phenomenological treatment of the question of Being qua Being started from this epistemological standpoint, with both these philosophers simply taking for granted the a priori validity of the truism of noncontradiction in knowledge claims about our world.[11] By contrast, Hartmann asserted that knowledge, when 'correctly defined', entailed the 'grasping of objects' and that such objects of knowledge, if they are to be grasped, must 'precede any effort to grasp them'.[12] 'Knowledge is precisely the being-in-relation of a consciousness to something-that-is'.[13] This includes the necessity for such 'objects of knowledge' to precede even our presupposed mastery of noncontradiction: 'being has the structure that it does, regardless of what anyone may think or know about it,' Predrag Cicovacki explains; 'whether the world as a whole is rational or not,

10 Nicolai Hartmann, 'How Is Critical Ontology Possible? Toward the Foundation of the General Theory of the Categories, Part One {1923}.' Trans. Keith R. Peterson. *Axiomathes* 22, no. 3, April 13, 2012, p.315–54. Hartmann, Laying the Foundations. Nicolai Hartmann, Possibility and Actuality {1938}. Trans. Alex Scott & Stephanie Adair, Berlin/New York, Walter de Gruyter, 2013. *Nicolai Hartmann, New Ways of Ontology* {1942}, Trans. Reinhard Kuln, New York, Routledge, 2012.

11 Author 2021 (Details omitted for double blind reviewing).

12 Roberto Poli, 'Chapter 1: Hartmann's Theory of Categories: Introductory Remarks', In *The Philosophy of Nicolai Hartmann*, Robert Poli , Carlo Scognamiglio, and Frederic Tremblay (eds), 1–32. Berlin: De Gruyter, 2011 p.2.

13 Hartmann, 'How Is Critical Ontology Possible?' Multiple citations of this article may be textually rendered as (HCOP).

purposive or not, cognizable or not'.[14] 'The problem [of Being] is always an ontological problem,' Hartmann (HCOP 316) also asserted. 'An epistemology which disputes this is hardly an epistemology. What it speaks of is then not "knowledge" at all'.

Hartmann started his philosophical career as a Marburg neo-Kantian, and his critical ontology began essentially as a reaction against what he considered to be his fellow neo-Kantian's misappropriation of Kant's critical philosophy. Rather than putting an end to metaphysics and ontology, a common assumption of neo-Kantianism, Hartmann argued that Kant's epistemological critique of the limitations of knowledge actually opened the way to a new, critically founded ontology. 'In asking about our epistemological limitations Kant was also asking a question to which philosophers had always assumed they knew the answer, namely, "How does thought relate to things?"'.[15] 'We have to take up the task of a new and more radical critique,' Hartmann argued, 'not only of 'pure reason' insofar as it harbours the a priori presuppositions of positive science—but a critique of the categorial formation of our ontological consciousness and overall consciousness of the world' (cited in INH 295). Central to this radical critique is the identification and correction of traditional metaphysical errors hindering philosophical progress. '[Kant's] critical philosophy is supposed to be distinguished from naïve or "dogmatic" philosophy in its determination to take the validity of no philosophical principle for granted'.[16] As is being argued in this essay, the very first metaphysical problem that needs to be addressed is the conflation of the application of noncontradiction to our world with the idea of it as an a priori truism. Even Kant, in his effort not to take anything for granted, seems to have been unable to appreciate this conflation: Kant took as the very starting-point in his attempt to scientifically reform metaphysics the a priori application of this truism beyond the phenomena, as supposedly the one thing we could rely upon about Being-in-itself, that the ontic constitution of reality

14 Predrag Cicovacki, *The Analysis of Wonder an Introduction to the Philosophy of Nicolai Hartmann*, New York, London, New Dehli, Sydney, Bloomsbury Academic, 2015, p.20.

15 Keith R. Peterson, 'An Introduction to Nicolai Hartmann's Critical Ontology', *Axiomathes* 22, no. 3, February 29, 2012. p.291–314, p.294. Multiple citations of this article may be textually rendered as (INH).

16 Thomas J. Bole III, 'Contradiction in Hegel's 'Science of Logic." *The Review of Metaphysics* 40, no. 3, 1987, p.515–34, p.518. Multiple citations of this article may be textually rendered as (CHSL).

could not possibly be self-contradictory.[17] This a priori starting-point was also never questioned either by Hegel or the neo-Kantians, or even Bohr and Einstein in their debate over the metaphysical implications of quantum discontinuity.

The specific problem that concerned Kant in the eighteenth century was the apparent incompatibility of Newton's causally governed world and Leibniz's monadological alternative.[18] '[It was] the antinomy of pure reason,' Kant explained, 'that first aroused me from my dogmatic slumber and drove me to the critique of reason itself, in order to resolve the scandal of the ostensible contradiction of reason with itself' (KRM 47). This dilemma was expressed in Kant's four cosmological antinomy,[19] and it also reflects the same basic problem confronting quantum physics with regard to the relationship between quantum discontinuity and the continuity of the physical world. Bearing in mind, the defining difference between Kant's antinomies, Newton and Leibniz's alternative accounts of the world, and the relationship between quantum discontinuity and the continuous causal structure of the physical world is that this latter antinomy is actually measurable, and it does define the limit of observable phenomena. Kant arrived at the separation of phenomena and noumena as the best way he could think of to account for the apparent contradiction between Newton and Leibniz's descriptions of the world. '[Attempts] to align Leibniz and Newton had created a tangle of contentious speculations that made metaphysics highly vulnerable to attacks from without,' de Boer (KRM 41) points out. 'As [Kant] came to see it, this tangle could be resolved only by cutting the umbilical cord between noumena and phenomena'. For Kant, it was the synthesis of pure understanding and sensibility that defined objective knowledge, differentiating between the phenomena of experience as the product of this synthesis, and the noumena that lay beyond any experience and beyond what it may be possible to objectively know. '[T]here are two stems of human cognition,' Kant (CPR

17 c.f., Terry Pinkard, 'Idealism', in *The Oxford Handbook of German Philosophy in the Nineteenth Century*, ed. Michael N. Foster and Kristin Gjesdal, Oxford University Press, 2015, p.3.

18 c.f., Karin de Boer, *Kant's Reform of Metaphysics: The Critique of Pure Reason Reconsidered*, Cambridge, United Kingdom; New York, NY, Cambridge University Press, 2020. Multiple citations of this book may be textually rendered as (KRM).

19 Immanuel Kant, *Critique of Pure Reason* {A1781/B1787}, trans. Paul Guyer and Allen W. Wood, Cambridge, Cambridge University Press, 1998, p.459-95. Multiple citations of this book may be textually rendered as (CPR).

A16/B30/135) argued, 'namely **sensibility** and **understanding**, through the first of which objects are given to us, but through the second […] they are thought.' By creating this division between phenomena and noumena, Kant was able to consign the ultimate source of such antinomy to the realm of the unknowable, beyond the limit of objective knowledge as defined by the synthesis of pure understanding and sensibility.

When it comes to the application of noncontradiction to our world, and particularly beyond the measurable phenomena, it is not sufficient simply to assert, as Bohr seems to have effectively done, that the answer is unknowable.[20] Because it is the logic itself of this principle as an a priori truism that is presupposed to validate it as an axiomatic law of knowledge, concluding that its initial source must be unknowable effectively validates the application of this truism, also, as a fundamental ontological law. This problem is exacerbated by the discovery that the very limit of measurable phenomena itself is defined by an antinomic relationship of continuity–discontinuity. Significantly, this means that the *application* of the truism of noncontradiction to our world as a fundamental ontological law can no longer be taken simply as self-evident. Regardless of how sure we can be that this antinomic relationship may represent the *appearance* of the *real* emergence of causality from randomness, the mere possibility of such a starting-point to our world means that the application of this truism beyond the phenomena can no longer be taken simply as an *a priori* certainty. As Aristotle originally pointed out, before we can presume to know anything about the world, we must first be satisfied with our mastery of noncontradiction as a fundamental ontological law. 'If a principle is such that anyone who is to know anything must grasp it,' Aristotle asserted with regard to the principle of noncontradiction, 'then the approach to any subject matter presupposes mastery of that principle'.[21] 'Aristotle says that without the principle of non-contradiction we could not know anything that we do know,' Paula Gottlieb (ANC) explains. 'Presumably, we could not demarcate the subject matter of any of the special sciences […] and the inability to draw distinctions in general would make rational discussion

20 Kant explicitly presupposed the application of the truism of noncontradiction to be self-evident.
21 Aristotle, *Metaphysics*, Trans. Hugh Lawson-Tancred, London, Penguin Books, 1998, p.87-88.

impossible'.[22] As the principle of noncontradiction serves, necessarily, as a fundamental ontological law in regard to any knowledge claims about our world, the need to take a stance is unavoidable with respect to how this law initially applies. By presupposing the application of the a priori truism of noncontradiction as a fundamental ontological law, Kant found himself compelled to consign the source of this truism to an unknowable realm of noumena. Understandably, this idea of a noumenal realm proved the sticking point for post-Kantian philosophers, such as Hegel, who naturally wanted to find a way beyond this supposed limitation. It also had a defining influence on Bohr, who, by contrast to Hegel and most other post-Kantian philosophers, embraced Kant's concept of noumena to account for the supposedly unknowable quantum realm, as has been well documented in the literature.[23]

If the application of the principle of noncontradiction within our world is initially defined by our world's ontic configuration, rather than simply as an a priori truism, as is almost universally assumed, then the object of a priori ontological analyses—that is to say, essentially this same ontic configuration—must almost certainly elude such analyses. The simple reason for this is that presupposing the application of this logical truism in our world also presupposes a *real* source for it, or, perhaps, to be more accurate, a solution to it. If the starting-point for our world is actually defined by the emergence of causality from

22 Gottlieb, Paula, 'Aristotle on Non-contradiction', *The Stanford Encyclopedia of Philosophy*, eds. Edward N. Zalta, Spring 2019.

23 c.f., C.A. Hooker, 'The Nature of Quantum Mechanical Reality', in *Paradigms and Paradoxes*, ed. R.G. Colodny, Pittsburgh, University of Pittsburgh Press, 1972, p.135–72. John Honner, 'The Transcendental Philosophy of Niels Bohr'. *Studies in History and Philosophy of Science Part A*, 13, no. 1, March 1982, p.1–29. David Kaiser, 'More Roots of Complementarity: Kantian Aspects and Influences', *Studies in History and Philosophy of Science Part A*, 23, no. 2, June 1992, p.213–39. Carsten Held, 'Bohr and Kantian Idealism', in *Proceedings of the Eighth International Kant Congress*, ed. H Robinson and G Brittan, Milwaukee, Marquette University Press, 1995. Steen Brock, *Niels Bohr's Philosophy of Quantum Physics in the Light of the Helmholtzian Tradition of Theoretical Physics*, Berlin, Logos, 2003. Karen Michelle Barad, *Meeting the Universe Halfway*, Durham, N.C., Chesham, Duke University Press, 2007. Hernan Pringe, 'A Transcendental Account of Correspondence and Complementarity' in *Constituting Objectivity: Transcendental Perspectives in Modern Physics*, ed. M. Bitbol, P. Kersberg, and J. Petitot, Berlin, Springer, 2009. Michael Cuffaro, 'The Kantian Framework of Complementarity', *Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics*, 41, no. 4, November 2011, p.1-28. Michel Bitbol, 'On Bohr's Transcendental Research Program' in *Niels Bohr and the Philosophy of Physics: Twenty-First-Century Perspectives*, ed. Jan Faye and Henry J Folse, London, Bloomsbury Academic, 2017, p.47–66. Plotnitsky, *Niels Bohr and Complementarity: an Introduction. Plotnitsky, The Principles of Quantum Theory, from Planck's Quanta to the Higgs Boson. Plotnitsky, Reality without Realism.*

randomness, then this contrary relationship would also have to *precede* our application of the truism of noncontradiction. In other words, there would effectively be no source or solution to this truism in our world, per se, because the very starting-point itself would be defined by the contrary relationship of causality–randomness. There would also be no need for an unknowable realm of noumena. As the initial starting-point for literally everything, such a *real* fundamental relationship would have to determine, *from the very outset*, how the principle of noncontradiction actually *applies* within our world. That is to say, in our all-encompassing world, the principle of noncontradiction could be expected *never* to apply simply as an a priori truism, but would have to function, *from the very beginning*, as a principle defined by *both* mutual exclusion *and* joint completion (i.e., complementarity). Bearing in mind that human cognition would have to operate, as well, from entirely within and as part of this same all-encompassing world. The end result is that, even in spite of the a priori truism of noncontradiction, of course, necessarily being an axiomatic law of logic in-itself, analyses that attempt to discern, a priori, the ontic configuration of our world without considering this potential limitation of the truism of noncontradiction are destined to keep, proverbially, tilting at windmills when it comes to the *real* fundamental configuration of our world.

Even though non-classical a priori analyses may question our classical understanding of this principle, they still invariably presuppose the inevitability of this truism in knowledge claims about the ontic configuration of our world. In other words, they still take the truism as an essential ingredient in their a priori analyses, and in the process, effectively discard the *real* object, that is, the inherent contrariness *itself* of our world's *ontic* configuration. This is what Hegel essentially did in his systematic collapsing of the distinction between object and concept.[24] In arguing that all things are inherently contradictory, Hegel did not invalidate the principle of noncontradiction, as many have presumed, but he did not actually get to its *ontic* core either. Hegel merely shifted the supposed source of

24 Alice A. Graves, 'Hegel's Doctrine of Contradiction', *The Journal of Speculative Philosophy* 22, no. 1/2, 1888, p.118–38. Thomas J. Bole III, 'Contradiction in Hegel's 'Science of Logic." The Review of Metaphysics 40, no. 3, 1987, p.515–34. Karin de Boer, 'Hegel's Account of Contradiction in the Science of Logic Reconsidered' *Journal of the History of Philosophy*, 48, no. 3, 2010, p.345–74. Luis Guzmán, *Relating Hegel's Science of Logic to Contemporary Philosophy*, London, Palgrave Macmillan, 2015.

contradiction from the ontic level and Kant's unknowable realm of noumena to the cognitive level and specifically the relationship between object and concept. The problem is made worse by the age-old presumption that the rules of thinking (i.e., logic) equate simply to the rules of Being. 'That [has given] logic an enormous predominance in metaphysics,' Hartmann (HCOP 318) explained, 'and if the problem of matter had not remained indigestible in the background like a bad conscience, it would have meant the complete hegemony of logic'. It is the possible emergence of causality from randomness as the *real* physical starting-point for our world that brings into question the absolute a priori validity of this assumption. Like Hartmann, Hegel's motivation was to extend Kant's critical philosophy by identifying and overcoming hidden presuppositions in Kant's original critique (CHSL 518); Like Kant, however, Hegel's epistemological approach failed to get beyond the conflation of the application of noncontradiction and the idea of it as a mere truism. The end result was that Hegel, even in arguably going further than Kant, was still unable to get to the *real* core of our world or of knowledge itself: Hegel, like Kant, did a thorough job of accounting for *how* the fundamental ontic configuration functions in our world, but he never actually managed to identify the *real* reason *why* it functions in the way that it does.

Because it is the initial starting-point itself for theorizing about our world that is in question, the answer must be found through an ontology that orients itself *hypothetically* toward the phenomena without presupposing the application of the truism of noncontradiction as an a priori certainty. 'In order to seize the Idea, a particular method was still required,' Hartmann (HCOP 319) explained, 'that of the "hypothesis", in which a critical reference back to the phenomena was clearly included'. Although Hartmann was referring here specifically to the Platonic theory of Ideas, the significance of this observation is still relevant today, particularly in our efforts to come to grips with the meaning of quantum discontinuity. 'This critical aspect of method, however, was lost over the course of time […] What remained was the dogma of the identity of the form of being and the *logos*,' (HCOP 319). It is asserted that to disregard the ontological implications of quantum discontinuity for our application of noncontradiction, and thus, potentially, for all theorising about our world, is to not understand the depth of the problem that this discovery represents. In our thinking, the initial

application of noncontradiction as a fundamental ontological law is not only taken for granted, but as asserted in this essay, is conflated with the idea of noncontradiction as an a priori truism. As a result, the real problem, if it is considered at all, has come to be assumed to revolve around whether or how we can ultimately have cognitive access to this initial application of the law of noncontradiction. It is this element of doubt raised by the discovery of quantum discontinuity, and particularly its metaphysical implications for our theorising about the world and, in the end, our conceptualisation of Being, that represents the real import of this discovery.

It is a procedural error to assume that the ontic meaning of quantum discontinuity and the doubt that it raises about our application of noncontradiction can be adequately investigated, a priori, through mathematics, logic (classical or non-classical), epistemology, or phenomenology. It has been taken as validated, more or less since Hegel, that logic, in providing the 'form and rules of thinking', can provide access to the 'absolute' configuration of Being: '[i]n Hegel, the *Science of Logic* is the immediate stage of the self-determining/unfolding of the absolute'. [25] 'Logic is the science of thought' Sebastian Rodl confidently asserts, echoing a common assumption in contemporary thinking. '[T]he science of thought is the science of what is in so far as it is. Logic is metaphysics'.[26] It is this straightforward assumption that the rules of Being must equate to the rules of logic that underpinned Einstein's position in his debate with Bohr, and it is this assumption that substantiates contemporary scientific and philosophical orthodoxy with regard to the ontic meaning of quantum discontinuity, Bohr being a notable exception, and has come to serve as the empowering mechanism behind ontological models that attempt to mathematize the world beyond the measurable phenomena. Even if this assumption about the equivalence of Being and thinking is essentially correct, the sticking point remains the almost universal taking of the *application* of the truism of noncontradiction to our world as merely an ontic certainty. While Frege and Russell, in laying the foundations for analytic philosophy, may have come to reject Kant's transcendentalism and Hegel's logic, they did similarly take for granted the application of noncontradiction to our

25 Clayton Bohnet, *Logic and the Limits of Philosophy in Kant and Hegel*, New York, Palgrave MacMillan, 2015, p.11-12.
26 Sebastian Rödl, 'Logic, Being and Nothing', *Hegel Bulletin*, 40, no. 1, 2018, p1–29.

world as self-evident and as *the* 'inner correspondence' connecting their own mathematized logic to our world. '[A] direct line of argument from Hegel's critique of the formality of logic to one of Frege's most fundamental insights,' can be traced, according to Stephen Kaufer: 'the basic unit of logical analysis is the "conceptual content" of a judgment'.[27] 'Kant recognised that the principle of non-contradiction provides a *sine qua non* for the intelligible use of concepts,' Kenneth Westphal explains, 'and thus a canon for cognitive judgment'.[28] 'Hence we must also allow the **principle of contradiction** to count as the universal and completely sufficient **principle of all analytic cognition**' (original emphasis) (CPR B191 280). The point is that the application of the truism of noncontradiction serves as a necessary ingredient for all such logical analyses, that is to say, not simply in its role as the first principle of logic per se, but before that, as the fundamental ontological law connecting all knowledge claims to our world. This is why Quantum discontinuity needs to be understood, *before all else*, as a critical ontological problem (i.e., first philosophy) because it brings into question the truism of noncontradiction as a fundamental ontological law. It is specifically this metaphysical function of noncontradiction, and its application to our world at the ontic level, that is almost universally taken for granted in our modern thinking.

## SECTION THREE: THE EMERGENCE OF CAUSALITY FROM RANDOMNESS AS THE LIKELY STARTING-POINT FOR OUR WORLD

Once it is appreciated that the application of the principle of noncontradiction to our world hinges not on its definition as a logical truism but on its *real* role as a fundamental ontological law, then it can be understood that the correct object of analysis with regard to the *ontological* significance of quantum discontinuity is not the extrapolated quantum objects, but the relationship defining the limit of measurable phenomena between this discontinuity and the continuous causal structure of the physical world. 'One of the oldest pieces of common philosophical wisdom is that the world is constructed of opposites, that its most

27 Stephen Kaufer, 'Hegel to Frege: Concepts and Conceptual Content in Nineteenth-Century Logic', *History of Philosophy Quarterly* 22, no. 3, July 2005, p.259-280, p.260.
28 Kenneth R. Westphal, 'Chapter 8: The Analytic of Principles' in *Kant*, ed. S. Baiasu and M. Timmons, New York, Routledge, 2021, p.3.

universal categories are polarities,' Hartmann (HCOP 349) reminded us. 'The endeavour to overcome this oppositionality […] is just as ancient'. 'Pretty much everyone agrees that beings and substance are composed of contraries,' David Reeve translates Aristotle as arguing in the *Metaphysics*; 'at any rate, they all say that the starting points are contraries […] and the starting-points of contraries are being one and being many'.[29] It is reasonable to suppose it more than mere coincidence that such a relationship (i.e., continuity–discontinuity) has been discovered, actually, to define the physical limit of measurable phenomena. The metaphysical significance of this relationship is almost universally underplayed because it has always been 'naturally' assumed that the traces of quantum interaction must, somehow, be caused by real quantum objects existing in space and time. 'The issue is not the existence of atomic objects as such (it is undeniable that something gives rise to the phenomena we observe),' Michael Cuffaro makes the supposedly self-evident observation, 'but whether our fundamental spatiotemporal and dynamical concepts are literally applicable to them'.[30] By taking for granted the applicability of the truism of noncontradiction, the issue becomes not the existence of such objects, which is presupposed to be self-evident, but the epistemological question of whether or how we can gain cognitive access to those objects. Although it is certainly true that something must give rise to the quantum interaction observed in scientific experiments, the real cause hinges *first* on our application of the principle of noncontradiction as a fundamental ontological law.

The sticking point has always been how to reconcile the enigmatic behaviour of quantum objects with the 'natural' presumption of a causal ontology. Again, the seminal example of this is the double-slit experiment and the particle–wave duality of quantum objects that this experiment appears to indicate. 'From a logical viewpoint, this was a paradox,' William Cropper explains. 'The duality seemed to be a threat, a "fundamental blemish" that might, if pushed too far, bring the entire theoretical edifice crashing down'.[31] Depending on the arrangement of this experiment and at which points the quantum objects are

29 Aristotle, *Metaphysics*, trans. C.D.C Reeve, Indianapolis, Cambridge Hackett Publishing Company, 2016, p.51.

30 Cuffaro, 'The Kantian Framework of Complementarity'.

31 William H Cropper, *Great Physicists : The Life and Times of Leading Physicists from Galileo to Hawking*, Oxford/New York, Oxford University Press, 2001, p.275.

observed, the resulting traces of quantum interaction (i.e., spatiotemporal discontinuity) correlate to either a particle-like 'machine gun' pattern or wave-like interference pattern. The specific threat that this discovery seems to pose to the mathematical structure of the physical world derives from the fact that it appears to bring into question the classical presupposition of a causal ontology. Following Galileo and Newton, and well before the discovery of quantum discontinuity, a causal ontology had come to be almost universally assumed. Even Kant and Hume presupposed the existence of such an ontology—and, of course, the necessary application of the truism of noncontradiction to our world. 'What they denied was that the human mind could have full access to this causality, and thus establish definitive causal connections between events'.[32] By the time Planck discovered quantum discontinuity, the presumption of a causal ontology had become so ingrained into our thinking, epitomized by the Newtonian belief in a mechanically clockwork universe, that the overwhelming expectation was that quantum objects should naturally have an independent and ultimately definable existence. In such an intellectual milieu, it is perhaps understandable, as well, that, when faced with the dilemma of quantum discontinuity, the prevailing perspective would tend to default to the epistemological question of how we can then know such objects, rather than the potentially more fundamental issue of actually coming to terms with the *real* status of quantum discontinuity as it *appears* in the phenomena. The imperative to maintain a causal ontology in the wake of the discovery of quantum discontinuity and the apparent threat such randomness poses to this classical ontology, and thus also presumably to the established mathematical structure of the physical world, hinges, again, entirely on the application of the truism of noncontradiction as a fundamental ontological law.

Once the hidden dogma behind these two age-old presuppositions is recognized—that is, a causal ontology and the truism of noncontradiction—then the relationship between spatiotemporal continuity–discontinuity defining the limit of measurable phenomena can be understood as representing simply the *appearance* of the *real* fundamental configuration of our world. It has long been accepted that the observable phenomena most likely represent the *appearance* of the *real* world. As the relationship between quantum discontinuity and the

32 Arkady Plotnitsky, *Niels Bohr and Complementarity*, p.13.

continuous causal structure of the physical world defines the acknowledged limit of measurable phenomena, it makes sense that such a spatiotemporal relationship also represents the *appearance* of the *real* fundamental configuration of our world, at least, that is, as this fundamental configuration would have to appear from within and as part of the same world. Bearing in mind, any such *real* randomness in this fundamental configuration would have to *appear* from within and as part of the same world as discontinuous in space and time, just as it's causal structure, again, from within and as part of the same world, would need to *appear* as spatiotemporally continuous. It is argued that the source of such antinomy remained an a priori problem *until* science was able to actually measure the limit of the phenomena together with the effective starting-point for knowledge itself and discovered this limit to be composed of an antinomic relationship between *spatiotemporal* discontinuity–continuity. This discovery effectively made the application of the principle of noncontradiction no longer merely an a priori truism but a *real* problem to be determined ontologically. The *spatiotemporal* quality of this relationship allows it not only to *precede* our application of noncontradiction, but actually to define the way this first principle of knowledge and logic should be applied in our world.

This essay considers causality and randomness to be *both* universal ontological categories, and also as combining to form *the* fundamental ontic configuration serving as the effective starting-point for our world. Causality is taken to be synonymous with the idea of the 'relation of cause and effect' while the principle of efficient causality is understood to represent the codification of this universal category. '[Galileo] defined efficient cause as the *necessary and sufficient condition for the appearance of something*,' (original emphasis): 'that and no other is to be called cause,' Galileo asserted, 'at the presence of which the effect always follows and at whose removal the effect disappears'.[33] Randomness, on the other hand, is defined simply as the absence of this relation (i.e., cause and effect); it is also the state of Being that is taken to have most likely preceded the existence of such causality. As a universal category, randomness is said to be 'intrinsic' in that it can still exist even if full knowledge of the state of a system is achieved;[34] in other words, its

33 Mario Bunge, *Causality and Modern Science*, New Brunswick, N.J., Transaction Publishers, 2009, p.33.
34 Manabendra Nath Bera et al., 'Randomness in Quantum Mechanics: Philosophy, Physics and Technology', *Reports on Progress in Physics*, 80, 2017, p.1–22, p.2.

existence is *complementary* to the relation of cause and effect in the phenomena. The resulting relationship of discontinuity-continuity would almost have to *appear* as mutually exclusive and jointly completing (i.e., complementary) from within and as part of the same world in order not to compromise the principle of efficient causality in that world. Although the rules of logic and mathematics can be taken as almost certainly equating to the rules of Being, it is important to understand that these rules still only equate to the *appearance* of the *real*: *they are not the real in-itself*. Again, the key argument of this essay is that this relationship between the spatiotemporal discontinuity-continuity at the limit of measurable phenomena is likely to represent the *appearance* of the fundamental ontic configuration of our world, not only as the starting-point for our world, but also as the necessary starting-point for *any* knowledge claims within and about our world, thus even contradiction itself. Without recognizing this possibility and actively taking it into account, a priori methods of analyses that presuppose the truism of noncontradiction as a fundamental ontological law invariably struggle to understand why the phenomenal limit of our world *appears* the way that it does.

When considered ontologically and without presupposing the truism of noncontradiction as the a priori starting-point for knowledge claims about our world or a causal ontology, the existence of quantum discontinuity points toward the emergence of causality from randomness, as the *simplest* explanation for the real configuration of our world. The use of 'simplest' here is intended to invoke Occam's Razor, that is, in its ontological sense as the principle of parsimony. 'Emergence theories presuppose that the once-popular project of complete explanatory reduction—that is, explaining all phenomena in the natural world in terms of the objects and laws of physics—is finally impossible'.[35] It is worth

35 Philip Clayton and Paul Davies, *The Re-Emergence of Emergence: The Emergentist Hypothesis from Science to Religion*, Oxford, Oxford Univ. Press, 2008, p.1-2.

reiterating that if the ontic configuration of our world does consist of the emergence of causality from randomness, then such *real* randomness would almost have to *appear* in the phenomena as discontinuous in space and time in order, actually, to *avoid* contradicting the established principle of efficient causality. It is worth restating, too, that such a relationship could only ever be measured from within and as part of this same contrary dynamic. In other words, the starting-point for human cognition would almost have to conform to this same fundamental ontic configuration of *both* causality *and* randomness. The use of discontinuity is preferred in this essay when referring to quantum interaction specifically because it emphasizes this defining aspect of *spatiotemporal* randomness. It is this quality of discontinuity in space and time that allows such randomness to *appear* in the phenomena as *complementary*—that is to say, *both* mutually exclusive *and* jointly completing—to the continuous causal structure of the physical world. The proposed starting-point for our world as the emergence of causality from randomness is logically consistent with the observable phenomena and *requires no deeper physical explanation.*

## SECTION FOUR: CONCLUSION

It is not the purpose of this essay to investigate the consequences of this error in the application of the principle of noncontradiction to our world but merely to point it out and to argue that it has only really become possible to discern this error because of the discovery of quantum discontinuity. By conflating the application of the principle of noncontradiction with the idea of this principle as an a priori truism, attempts to understand, a priori, how noncontradiction applies in our world invariably presuppose, from the outset, the validity of this truism with regard to knowledge claims about our world. It is from this standpoint that such analyses then attempt to discern how this truism ontically functions in our world. This essay argues that the *simplest logical* explanation for the existence of quantum discontinuity is the emergence of causality from randomness as the effective starting-point for our world. It is reasonable to conclude that the truism of noncontradiction, in-itself, *never* actually applies in our world, *even at the ontic*

*level or as a fundamental ontological law*. The relationship of causality–randomness can be understood not only to *precede* the formation of our world and the application of the truism of noncontradiction, but also to define the fundamental ontic configuration of our world, including as the starting-point for any knowledge claims about our world. This can include, also, how the principle of noncontradiction should be applied within our world, even at the most fundamental ontic level. If the relationship between quantum discontinuity and the continuity of the physical world can be understood to represent the *appearance* of this *real* configuration, that is, from within and as part of the same world, then it is reasonable to suppose, as well, that this relationship can serve to define how the principle of noncontradiction should be applied in our world.

Previous iterations of this argument have been criticized and rejected because they were judged not to have sufficiently considered logic and particularly the literature on quantum logic. It is asserted in this essay that the *ontic* meaning of quantum discontinuity is, *in the first instance*, a matter for *ontology*, not logic, mathematics, epistemology, or phenomenology. This is because such a discovery actually brings into question the very starting-point itself for any knowledge claims about our world and specifically the classical application of the truism of noncontradiction as an a priori fundamental ontological law. The first step in metaphysics—indeed, in any thinking about our world—must be to determine the initial starting-point and connection for knowledge within our world. This needs to be carried out without initially assuming the validity of the application of *any* principle, including the a priori truism of noncontradiction even as an axiomatic law of logic itself. Although the physical laws of nature are certainly the domain of physics, the ontic foundation for those laws, and particularly the clarification of our mastery of the principle of noncontradiction, is a question that can only, in the end, be addressed ontologically—it is not merely a problem of measurement, or of logic, or even of mathematics. Based on the measurable phenomena, it is likely that the 'quantum level' *is* 'unthinkable' *because* it is defined by randomness as a universal ontological category, in other words, it is defined by the absence of *any* relation of cause and effect, the presence of which is literally necessary for such thinking to be possible, and as part of *the* fundamental relation that comprises, *along with causality*, the ultimate ontic configuration and effective starting-point for our world.

The purpose of this essay has been to introduce the novel idea that the initial

application of noncontradiction as a fundamental ontological law connecting all knowledge claims to our world is most likely linked to the relationship between quantum discontinuity and the continuous causal structure of the physical world as defining the limit of measurable phenomena. Based on this observable relationship, the simplest explanation for the ontic configuration of our world, and thus also the starting-point and ontic connection for any knowledge claims within and about our world, is the emergence of causality from randomness. It is argued that quantum discontinuity most likely represents traces of this, so to speak, primordial randomness, the existence of which is likely to have preceded not only the causal structure of our world but also its very emergence and formation. The key to such a proposal is *not* to take the initial application of the truism of noncontradiction as an a priori certainty and to appreciate that it is, in fact, logical for such spatiotemporal randomness to precede the principle of efficient causality in our world. The novelty of this argument, the profundity of its implications, and the fact that it brings into question the very foundation itself connecting any knowledge claims to our world means that it demands careful consideration.

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