JEAN-LUC NANCY AND THE FRACTAL ONTOLOGY OF NATURE

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ABSTRACT: This paper seeks to elucidate an affinity between fractal geometry and the ontology of Jean-Luc Nancy. I argue that Nancy's notion of being singular plural, whereby each being is defined as a circulating play of other beings exposed to one another in the space of a sheer opening—what he calls "being-with" or the spacing that allows for beings to be together and relate to one another—provides a description of existence which is both ecological and fractal. It is ecological inasmuch as every being now appears as a temporarily cohering system based on stable networks of circulation thus marking them as fundamentally processual and temporal rather than substantive. It is fractal insofar as each being appears as a composite of a circulation of other beings who, themselves, are also composites of other circulating beings and so on and so forth both up and down scale levels. To substantiate these points, Nancy is brought into conversation with Mandelbrot and the different ways in which fractals have been used to model ecological phenomena to show that these two descriptions are consonant with one another. Moving from the significance of how a fractal is defined as a certain kind of roughness and excess within geometric space to the importance of, specifically, self-affine fractals in the modeling of natural formations like mountain ranges to the aggregations of Brownian motion, this paper seeks to elaborate upon a fractal ecology of nature by way of Nancy and Mandelbrot.

KEYWORDS: Phenomenology; Ecosystem; Fractals; Ontology; Scaling

It is the goal of this paper to clarify some points of connection between Jean-Luc Nancy's ontology and fractal geometry to show in what way, as Nancy might say, fractal geometry is able to *expose* the world according to the structures of being-with, spacing, and circulation that Nancy theorizes. Accordingly, the intent is to show that Nancy's ontological picture is, in essence, both fractal and ecological and that these two terms co-constitute one another. It is precisely because of Nancy's fractal ontology that every being can be conceived of as *both* a part of an

ecology (circulating within it) as well as an ecology in its own right (a "discrete and transitory" cohesion produced by its own circulation). As we shall see, all of these ecological cohesions will show themselves to be always already affected and in contact with all the rest as well as in a state of tension in the sense that every cohesion could be otherwise and, in its needing to react to that which affects it, is always in the act of reconstituting itself and its circulations in such ways that the cohesion could be lost or significantly changed. Put otherwise, these circulatory cohesions will be understood to have *time* as an essential element of their always transitory constitution.

NULLITY AND CIRCULATION

In Nancy's work, multiple themes and concepts show themselves again and again, repeat themselves, iterate with slight differences, and circulate. This structure already performs his philosophical theses. As opposed to the construction of a single, monolithic discursive system that would reduce all things to a discrete ordering within a single *magnum opus*, Nancy wishes to foreground something else, something which cannot be contained: a nullity which opens up space and time as fields. In focusing on the excessive—that which surges forth, that which surprises, that which opens space—we find his work to focus on a necessary movement of circulation since there is no possibility of containing things within a stable and ordered system of essences. As he tells us in a sort of a summary of the tenor and inclination of his project, "if nihilism corresponds to the accomplishment of onto-theology according to the logic of a 'bad' infinite of presupposition…a thinking of 'creation' constitutes the exact reverse of nihilism, conforming to the logic of a null presupposition (which is equivalent as well to a 'good' infinite, or actual infinite)."[1] This "logic of the null presupposition" which Nancy identifies with the good infinite functions as the *condition* for the production of beings and existence as such. It is the first surging forth of an empty field into which all else will be able to appear and make itself rather than the annihilating return to voidness associated with nihilism. Here, we approach his central themes of touching, exposure, spacing, circulation, and being-with.

For Nancy, all of these terms describe a certain fundamental ontological condition in which every being which is, *exists* as itself insofar as it exists *in the open*

[1] Nancy (2007, p. 71).

space with other beings. Hence, we can arrive at the meaning of the phrase he gives us as the title of one of his works, *Being Singular Plural*. As he tells us within that work, "Being singular plural…mark an absolute equivalence, both in an indistinct *and* distinct way."[2] But what does it mean to say that these three terms form an "absolute equivalence"? As he clarifies further, "Being does not preexist its singular plural…Being absolutely does not *preexist*; nothing preexists; only what exists exists…[and] That which exists, whatever this might be, coexists because it exists…the world is the coexistence that puts these existences together…*Being singular plural* means that the essence of Being is only as co-essence."[3] Being is always already singular *and* plural. Moreover, because of this, there can be no relation of priority or causality set between these terms. There is not, first, being and then plurality, and then singularity, or, first, singularity, and then being, and then plurality. There is only the simultaneity of a single moment and a single opening in which Being comes to be as itself within existence only insofar as that existence is already plural, which precisely means, made up of singulars. This is why Nancy tells us that "[t]he essence of Being is the shock of the instant [*le coup*]…a lash, blow, beating, shock, knock, an encounter, an access."[4] It happens *all at once*. And yet this happening, and this opening, which is always already *all at once* in an indissociable conjunction of *being singular plural* can be divided. It is just that it cannot be divided before its coming into existence. They come into existence *at once* together, but what comes into existence here is a differentiated multiplicity so that one may also add that the togetherness that comes into being is split or inoperative, that is, *not* together.

This is one of the necessary aporias that we find in Nancy, that is, that beings are with one another only insofar as they are separated from one another. If there were no separation, they would not be *together* but would have *fused* into each other. Spacing and separation is the necessary condition of community itself, thus he can describe for us how "[t]he togetherness of singulars is singularity 'itself.' It 'assembles' them insofar as it spaces them; they are 'linked' insofar as they are not unified."[5] Here, he emphasizes for us how much these concepts undo

[2] Nancy (2000, p. 28).
[3] Nancy (2000, pp. 29-30)
[4] Nancy (2000, p. 33)
[5] Nancy (2000, p. 33)

themselves. The singularity is itself, not because it is separated, but because it is together, and yet, this togetherness is what it is precisely because the linking between them cannot unify them; they are assembled together only insofar as they are spaced out from one another. There is, at once, togetherness *because* of separation as well as separation *because* of togetherness.

This is the structure of what Nancy calls being-with and it is, for him, the most basic structure of existence. As he tells us, "[a]ll ontology can be reduced to this being-unto-self-unto-others…essence *is* nothing more than the exposition of its subsistence: the exposed face of what subsists, existing only insofar as it is exposed."[6] Or, elsewhere, "[e]ssence is in itself existence…Essence exposes *itself*."[7] What this makes clear for us is that Being is always already being-with precisely because Being can only come to be *as existence* where this existence is understood to mean the exposure of beings to one another. That is, I exist only insofar as I am exposed to others in a position that I take up, and my essence is in this very exposition of myself to others through which meaning is constructed and created in a circulation between us.

Accordingly, Being, which is always being-with, is always already spatial (and temporal). Nancy describes this at length to us:

> The very simplicity of 'position' implies no more, although no less, than its being discrete, in the mathematical sense, or its distinction *from*, in the sense of *with*, other (at least possible) positions, or its distinction *among*, in the sense of *between*, other positions. In other words, every position is also dis-position, and, considering the appearing that takes the place of and takes place in the position, all appearance is co-appearance.[8]

So Being, as existence, is the plurality and multiplicity born of beings taking up positions that exposes each one to all the rest; it is the position, the dis-position, and the ex-position. And it is this very exposure of one to the other that Nancy characterizes as the sharing out of Being. That is, Being is shared, not because we are all alike in the sense of having the same quality (we all have hearts, for instance) but, rather, what we share is a condition that marks each one of us as radically distinct from one another—each one their own "origin of meaning"[9]—

[6] Nancy (1991b, p. 4)
[7] Nancy (1991b, p. 3-4)
[8] Nancy (2000, p. 12)
[9] Nancy (2000, p. 85)

such that we must be in contact with one another and expose one another. What we share is existence, where this existence is each time separate and distinct from all other existences while being, like them, exposed and singular.

Nancy illustrates for us how much this ontology amounts to "the banal phenomenology of unorganized groups of people"[10] as when he gives us the example of the train compartment where he says, "[p]assengers…are simply seated next to each other in an accidental, arbitrary, and completely exterior manner. They are not linked. But they are also quite together inasmuch as they are travelers on this train, in this same space and for this period of time."[11] And this example helps to clarify for us how much what Nancy is getting at is "an ontology of bodies."[12]

However, it is not just that there would be a mass of differentiated and disorganized bodies hitting up against one another for it is this contact between these bodies and the circulation and movement between them that Nancy characterizes as the way in which meaning occurs. It is precisely through the exposition of beings with one another, that meaning forms itself. Accordingly, Nancy says that "*Being itself is given to us as meaning.* Being does not *have* meaning. Being itself, the phenomenon of Being, is meaning that is, in turn, its own circulation—and *we* are this circulation."[13] We find here yet more terms conjoined. Just as the *with* of being-with is not added onto Being but is always already part of Being, and just as we saw earlier that there is an equivalence between plurality, singularity, and Being itself, so too here do we find something similar going on. Being, whose meaning is found in the exposition of existences to one another, is *also* productive of meaning in general precisely because this process of exposition is that through which meaning becomes produced in the circulation that takes place in the spacing out between beings.

We might summarize Nancy's ontological picture like so: Being happens, in a single blow, as a multiplicity of existences taking up positions in space, each exposed to the other, where this very exposition of each to the other motivates, between them and within this opening of space, a circulation which itself is

[10] Nancy (1991b, p. 7)
[11] Nancy (1991b, p. 7)
[12] Nancy (2000, p. 84)
[13] Nancy (2000, p. 2)

productive of meaning precisely because what meaning is *is* this exposition in which each being is in contact with the others while remaining separated from them since they cannot fuse together and this never-ending circulation of meaning between spaced out beings corresponds to the dimension of time. Being is opened up within existence as space, time, and meaning because of the impossibility of beings being anything other than *both* singular and plural which corresponds to their always already being in contact with one another at the same time as this contact is not capable of ever reaching a fusion and, therefore, an endpoint.

However, just because a final fusion cannot be made does not mean that certain circulations cannot be stabilized as temporal cohesions within this opening, and it is here that we can look more closely at Nancy's notion of the singularity and what it implies for this picture of beings existing within the spacing out of Being in which they are always already with one another as a non-totalizable plurality of singular beings exposed to one another where this very exposure produces a circulation of meaning *between* them that can never be closed down or completed by virtue of the structure of this ontological constitution.

As we mentioned before, Nancy's ontology is an ontology of bodies where he specifies for us that "bodies" does not just refer to human bodies or animals but to "every body, whether they be inanimate, animate, sentient, speaking, thinking, having weight, and so on. Above all else, 'body' really means what is outside, insofar as it is outside, next to, against, nearby, with a(n) (other) body, from body to body, in the dis-position"[14] and, furthermore, a "singularity is always a body, and all bodies are singularities (the bodies, their states, their movements, their transformations)."[15] What this specifies for us is the fact that Nancy's ontology can be understood as a certain kind of materialism that is based on the exposition and contact that occurs between actually existing bodies at their limit surfaces where they touch one another. It is this very process of contact, touching, and movement that is itself significant in the development of his concept of singularity and its meaning. If a singularity is always a body, this does not mean that it is always a body in how we might ordinarily think of the term. As we just saw,

[14] Nancy (2000, p. 84)
[15] Nancy (2000, p. 18)

Nancy tells us that a singularity could be the state, movement, or transformation of a body forming its own body or singularity.

It is here that we can get a sense of just how radical Nancy's ontology is. When he refers to beings or singularities that are exposed to one another at their limits and their production of meaning between one another, he means this in a very literal way that provides us with a granular and scaling picture of existence. This can, perhaps, be most clearly seen when he distinguishes the singularity from the individual. As he tells us:

> the essence of singularity…is not individuality; it is, each time, the punctuality of a 'with' that establishes a certain *origin of meaning* and connects it to an infinity of other possible origins. Therefore, it is, at one and the same time, infra-/intraindividual and transindividual, and always the two together. The individual is an *intersection of singularities*, the discrete exposition of their simultaneity, an exposition that is both *discrete and transitory*.[16]

The singularity establishes an origin of meaning produced through its exposition to the other possible origins. The reason why the individual can be characterized as an "intersection of singularities" is because the individual is not unitary or finalized, it is both "discrete and transitory" where this characterization expresses both the individual's cohesion and the fact that this cohesion is momentary and the production of something else, namely, this very intersection of singularities occurring by means of their circulation. At the same time, this "discrete and transitory" cohesion of singularities in the individual forms its *own* origin of meaning capable of exposition. This is why Nancy can tell us that "singularity is not an identity…But identity, whether individual or collective, is not a sum total of singularities; it is itself a singularity."[17] In other words, what a singularity designates is *that which can be exposed*, and that which can be exposed can be itself a discrete and transitory composition of other sites of exposition. A singularity is made up of singularities *and* makes up singularities. This is why the individual or collective, while being composed of singularities, is itself *also* a singularity.

Hence, we find that Nancy's descriptions do not limit themselves to only humans or animals, but encompasses a broad array, as when he tells us that:

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[16] Nancy (2000, p. 85; emphasis mine)
[17] Nancy (1991b, p. 7)

> this circulation goes in all directions, at once, in all the directions of all the space-times [*les espace-temps*] opened by presence to presence: all things, all beings, all entities, everything past and future, alive, dead, inanimate, stones, plants, nails, gods—and 'humans,' that is, those who expose sharing and circulation by saying 'we,' by saying to themselves in all possible senses of that expression, and by saying we for the totality of all being.[18]

As should be clear, the singularity is not anthropocentrically limited (even if the "human" plays the role of "saying we for the totality of all being" which Nancy characterizes as their exposition of the conditions of exposition, that is, "the meaning of Being…which *we* make explicit, we 'humans,'…for the totality of beings."[19] Accordingly, he can ask such a question as "[w]ould not stones, mountains, the bodies of a galaxy be 'together' seen from a certain perspective not ours?"[20]

Although he leaves such a question unanswered, it would be hard to argue that it could be answered in any but an affirmative way given what we have already seen from Nancy and his further characterizations of existence, exposure, and singularity such as his explanation that the "difference between humanity and the rest of being…does not distinguish true existence from a sort of subexistence. Instead, this difference forms the concrete condition of singularity. We would not be humans if there were not 'dogs' and 'stones.' A stone is the exteriority of singularity in what would have to be called its mineral or mechanical actuality."[21] If this is the case, then we can grasp the idea that even if we grant to Nancy this role for humans as those who *expose* the world, this does not thereby grant them any central status and, in fact, the world would continue to circulate in its being-with without us. For Nancy, "'humanity' is not the subject of the world…it is not its origin or end. It is not its meaning; it does not give it meaning. It is the exponent…it exposes the world and its proper being-with-all beings in the world."[22] Accordingly, even where there are no humans, "the rose grows…because it grows along with the reseda, the eglantine, and the thistle."[23]

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[18] Nancy (2000, p. 3)
[19] Nancy (2000, p.12)
[20] Nancy (1991b, p. 6)
[21] Nancy (2000, p. 18)
[22] Nancy (2000, p. 85)
[23] Nancy (2000, p. 86)

However, where there are humans, not only are humans exposed, but also the singularities they create. In sum, as Nancy tells us, "I say 'singularities' because these are not only individuals that are at stake…Entire collectivities, groups, powers, and discourses are exposed here, 'within' each individual as well as among them. 'Singularity' would designate precisely that which, each time, forms a point of exposure, traces an intersection of limits on which there is an exposure."[24]

Here, we can finally begin to think of what mathematics might expose for us in conjunction with this ontological picture drawn by Nancy. Thinking of this idea of the singularity as a structure that is made up of singularities and which composes greater singularities and the fact that the singularity as such, is always already defined by its being within a plural circulation with other singularities in open space, we can begin to see that what Nancy has described for us is a single structure (the circulation of beings in the *with*) that repeats itself at a variety of different scale levels (from the rock to the galaxy) where each iteration of itself (such as an individual) is always to be understood as composed of structural copies of itself (the intra-, infra-, and transindividual singularities we saw earlier, for instance). In other words, Nancy has described a fractal ontological structure of existence where the singularity roughly corresponds to the fractal.

IRREDUCIBLE ROUGHNESS AND THE LIMIT OF THE SMOOTH

As Mandelbrot tells us, "I coined *fractal* from the Latin adjective *fractus*. The corresponding Latin verb *frangere* means 'to break:' to create irregular fragments. It is therefore sensible…that, in addition to 'fragmented'…*fractus* should also mean 'irregular.'"[25] Accordingly, a fractal can be thought of as an irregular, broken off fragment of something else. As we will see, however, its "irregularity" will, nevertheless, have a certain kind of order which is why Mandelbrot describes fractal geometry as "a workable geometric middle ground between the excessive geometric order of Euclid and the geometric chaos of general mathematics."[26] But what, precisely, does it mean that it forms a middle point between these two extremes while nevertheless appearing as irregular? One answer to this question

[24] Nancy (1991b, p. 7)
[25] Mandelbrot (1983, p. 4)
[26] Mandelbrot (1989, p. 3)

is roughness.

As Mandelbrot describes it to us, a standard Euclidean geometric shape is typified by its smoothness, that is, if you:

> Take a geometric shape and examine it in increasing detail…tak[ing] smaller and smaller portions near a point P, and allow every one to be dilated, that is, enlarged to some prescribed overall size…the enlargements become increasingly smooth. Ultimately, nearly every connected shape is locally linear. One can say, for example, that 'a generic curve is attracted under dilations' to a straight line…And 'a generic surface is attracted by dilation' to a plane…More generally, one can say that nearly every standard shape's local structure converges under dilation to one of the small number of 'universal attractors'.[27]

In other words, as you zoom in, everything becomes straight, smooth, and flat; every object's local structure can be reduced to one of these "universal attractors." However, as Mandelbrot immediately notes after this description, "[n]ature *fails* to be locally linear."[28] The Euclidean shape which can always be reduced to a linearity upon enlargement cannot actually appropriately model formations in nature. Instead, something else happens. The jagged shape, rather than smoothing itself out as I zoom in on it, stays rough, maintains its jaggedness. Which leads us to one of Mandelbrot's definitions of fractals as "shapes whose roughness and fragmentation *neither* tend to vanish, *nor* fluctuate up and down, but remain *essentially unchanged* as one zooms in continually and examination is refined. Hence, the structure of every piece holds the key to the whole structure."[29] The repetition and self-similarity that tend to come to mind first when we think of fractals is here tied up with this idea of roughness. The fractal shape is rough precisely because it does not reduce itself to smooth parts as one zooms in on it, instead, it repeats itself and its roughness in a continuing iterative process as one scales it. It is, to be sure, *also* self-similar (or self-affine) which Mandelbrot describes as expressing that "each part is a linear geometric reduction of the whole"[30] but it is this self-similarity that is productive of its irreducible roughness. That is, it is rough because it is self-similar.

A classic example of the application of this is in Mandelbrot's essay on

[27] Mandelbrot (1989, p. 3)
[28] Mandelbrot (1989, p. 4)
[29] Mandelbrot (1989, p. 4)
[30] Mandelbrot (1989, p. 4)

measuring the coast of Britain. As he tells us there:

> It is evident that the length is at least equal to the distance measured along a straight line between its beginning and its end. However, the typical coastline is irregular and winding, and there is no question that it is much longer than the straight line between its end points…[thus] coastline length turns out to be an elusive notion that slips between the fingers of the one who wants to grasp it. All measurement methods ultimately lead to the conclusion that the typical coastline's length is very large and so ill determined that it is best considered infinite.[31]

Although the idea that coastline of Britain should be considered infinite may seem ridiculous at first, what is meant to be expressed by this is that the coastline has an irreducible roughness that cannot be fully smoothed out by us in our attempts to measure it. Although we can, of course, measure the coast of Britain for a variety of technical and practical purposes, we cannot say that we *know* the exact and precise length of the coast, and this is because of its fractality. As Mandelbrot clarifies further, "[w]hen a bay or peninsula noticed on a map scaled to 1/100,000 is reexamined on a map at 1/10,000, subbays and subpeninsulas become visible. On a 1/1,000 scale map, subsubbays and sub-subpeninsulas appear, and so forth. Each adds to the measured length."[32] As one zooms in closer to the coast to have a more precise measurement, more length is added each time as more structures become apparent that need to be measured. Unlike the "coast" of a manufactured in-ground pool, the coast of Britain is too irreducibly rough to be measured *completely* by human instruments, it exceeds us and our attempt to grasp it.

What this failure makes apparent for us is precisely this disconnect or non-coincidence between the measuring instrument and that which it seeks to measure; the insufficiency of an ideality to measure reality. Of course, the measuring instrument is itself an object that is modeled on ideal Euclidean shapes. If we think of a ruler, we think of a rectangle and a straight line. In seeking to measure a coastline, what we are trying to do is make straight lines that are sufficiently small as to be equivalent to the coastline, however, as we noted above, this is precisely what is impossible in the fractal form since that would amount to its being locally linear and capable of a final smoothing out. In other words, the

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[31] Mandelbrot (1983, p. 25)
[32] Mandelbrot (1983, p. 26)

rough coastline cannot be made into a sequence of very small straight lines; it cannot be appropriately covered by them. There is always something that is left over whether it is a piece of the coastline which exceeds it or a piece of empty space that the straight line must introduce to smooth it out; a coincidence between them is impossible.

This brings us to a more technical definition of the fractal that clarifies its distinction from Euclidean shapes and its irreducible roughness and non-coincidence. As Mandelbrot defines it for us "*[a] fractal is by definition a set for which the Hausdorff Besicovitch dimension strictly exceeds the topological dimension.*"[33] This is in contrast to Euclidean shapes where the Hausdorff Besicovitch dimension is *always* precisely the same as the topological dimension (and always an integer). Although there are some fine technical differences between the Hausdorff Besicovitch dimension and the fractal dimension, for our purposes, they can be considered to be equivalent to one another and to measure roughness, or complexity, understood through the filling up of space. As Kenkel and Walker put it "[f]ractal dimensions…quantify the degree to which the trace 'fills' the plane."[34] And this goes along well with a description given by Frame and Urry where they tell us that "[f]or a shape that is broken or rough, as we look more closely we see that its jaggedy edges fill up some, but not all, of the space around the shape. We'd like to measure how the fraction of the space occupied by the shape scales with how closely we look."[35] In other words, the fractal dimension quantifies the degree to which something is in excess of that which seeks to cover it up according to a smooth reduction. We can see why the alternate name for a topological dimension is a Lebesgue *covering* dimension. Accordingly, we can say that for a Euclidean object, the reason why the topological dimension and the fractal dimension coincide is because the object is capable of being fully covered or fully reduced to smoothness without producing an excess. There is here a possibility of total coincidence.

Continuing with Frame and Urry's description of a fractal dimension, they go on to tell us that "[t]he higher the dimension, the more completely a shape's folds, branches, and wiggles fill the space that surrounds it. Or put another way, the

[33] Mandelbrot (1983, p. 15)
[34] Kenkel and Walker (1993, p. 54)
[35] Frame and Urry (2016, p. 56)

higher the dimension, the higher the roughness, the higher the complexity."[36] That is, the fractal shape, because of its roughness and its non-coincidence with its topological dimension, has a fractal dimension that measures its exceedance from its topological dimension. However, as Frame and Urry's description makes clear, the fractal dimension is *higher* as the fractal object fills in more space and approaches a coincidence. Here we can see something paradoxical going on. To the extent that something is fractal, it is in excess of its topology. However, after having already exceeded itself, it can, nevertheless, continue to roughen itself further to try and fill up that space. At the same time, this attempt to fill up more space through a roughening only increases its complexity and the density of its jaggedy edges and lines. In this sense, one might say that the fractal dimension functions as an inverse measurement of a fractal object's spacing out or a direct measurement of its circulation seeking to fill in that spacing. Either way, what gives the fractal object its character is the impossibility of a coincidence in which this space would be eliminated through a complete covering in a crossing over in which we would reach the ideality of a Euclidean object in which the topological dimension and the fractal dimension are the same. No matter how much it roughens itself up, the fractal object, which is always already within an exceedance with respect to its topology, cannot return to a dimensional coincidence since this can only happen through a simplification of itself down to a smoothness.

By now the affinities with Nancy should be clear. One might say that as soon as you have a fractal, you have the opening of space and the *with* which corresponds to this very excess of the fractal with respect to topology. This is what it means to be a fractal, that one is in exceedance. After having found oneself within this exceedance in which a spacing occurs, circulation becomes possible and can occur at lower and higher levels so that what the fractal dimension can be seen to quantify is the level of circulation occurring within a fractal singularity.

Here we can begin to connect these two concepts of the fractal and the singularity. If the singularity is, as we have already discussed, nothing more than a site of exposure and contact which is produced through a circulation of other singularities within it and implicated in the composition of singularities above it,

[36] Frame and Urry (2016, p. 58)

then a singularity can be thought of as that which repeats itself at all scale levels according to a logic of circulation within an opening of space. As we have just seen, this structure can be fractally modelled since the fractal is that which is produced through the opening of empty space, in the exceedance of its topological dimension, producing a variety of different circulations of itself within this space corresponding to its fractal dimension and, therefore, its complexity and roughness. Moreover, the fractal's composition is typified by its being composed of pieces of itself which would have the same structure as it. Just as a singularity is a site of exposure that is produced by means of a circulation of other sites of exposure, so too is a fractal rough only insofar as it is produced by means of a circulation of other fractals within it. And just as Nancy argues that this opening of the space of the *with*, in which the exposition of beings to one another takes place, is also, by that very fact, the opening of time, so too has it even been argued that "time may be an emergent property" arising from a fundamental fractal structure to the universe, that is, to a fundamental dimensional non-coincidence that opens it up.[37]

Unlike the picture that Nancy gives us, however, in which the circulation of Being produces singularities which are transitory, one might argue that fractal geometry gives us a structure which is, in its own way, limiting or constricted. Rather than a free circulation of beings in which contingent and transitory singularities would be produced as momentary cohesions, we would have an invariant repeating structure. That is, if the fractal is that which is rough because it is self-similar and thus composed of perfect copies of itself *ad infinitum* then how is this not just another way of representing the single being that Nancy tells us is "a contradiction in terms" since it would be "its own foundation, origin, and intimacy…[thus] incapable of *Being*"?[38] As we will see in the next section, fractals *can* model the sort of circulation of Being that Nancy describes for us, but it requires that we leave self-similar fractals in favor of self-affine ones.

SELF-SIMILARITY, SELF-AFFINITY AND (IN)CALCULATION

Although, popularly, fractals are often represented as being invariantly self-

[37] Frame and Urry (2016, p. 85)
[38] Nancy (2000, p. 12)

similar with themselves, this, in fact, denotes only one kind of fractal structure. As Mandelbrot specifies for us, the self-similar fractal is that which is not only composed of "linear geometric reductions" of itself, but also that in which these reductions maintain a constant ratio in *all* directions.[39] What this relates to is distortion and variance within the fractal itself. That is, it is because a self-similar fractal maintains the same reduction ratios in all directions that it can be scaled without any distortion or variance from one magnification level to another. The self-similar fractal does not change as one zooms into it; it embodies perfect, lossless iteration. This is why, in principle, one could zoom into a self-similar fractal forever without noticing any difference, only a repeating pattern.

Although self-similarity of this ideal type may be possible in mathematics, it does not actually appear in nature. As Kenkel and Walker remind us:

> For natural objects, the elegant self-similar property of mathematical fractals does not apply, just as we do not expect to find true Euclidean objects (circles, squares). However, many natural objects (e.g. coastlines, ecological habitats and landscapes) do display some degree of 'statistical' self-similarity, at least over certain spatial scales (statistical self-similarity implies a scale-related repetition of overall complexity, but not of the pattern itself).[40]

If natural objects are not, in fact, self-similar in this ideal mathematical sense that we find in geometry, then, in what sense are they fractal? As just mentioned, rather than a perfect iteration of one pattern, what you find is a "statistical" self-similarity which corresponds to a repetition, not of one pattern, but of overall complexity. Across a certain range of scales (often 100-fold magnification or more[41]) you find that what is repeated is a certain distribution of values, but not the *same* values. In other words, there is *both* repetition and difference here. The natural fractal iterates and repeats itself, but it does so in a way in which each iteration is not a perfect copy of the others, even if it is similar or related to them. It is both the same and different.

Another name for these "statistical" self-similar fractals is self-affine. And here, we can briefly note the interesting difference between the choice of words. Whereas "similar" is etymologically related to the Latin *similis* which can be interpreted as "like," "affine" is related to the Latin *affinis* which can be

[39] Mandelbrot (1989, p. 4)
[40] Kenkel and Walker (1993, p. 54)
[41] Frame and Urry (2016, p. 56)

interpreted as "bordering on." The difference implied is that between the simultaneity of two beings crossing over one another in near identity versus the spacing out of beings which are close to one another and in contact with one another without a crossing over. To be affine is to be at the border and in touch; the exposition of surfaces to one another. As related to one another, but not the same as one another, the circulation takes on a certain level of unpredictability even though, in another sense, it is still predictable. That is, we know *that* a certain distribution of values will repeat itself, but we do not know *what* those values will be until it happens. Moreover, we know that at a certain point this circulation will break down.

As Frame and Urry make clear for us, "the atoms making up a fern do not look like little ferns. Also, nature is messy and complicated. Nothing grows in complete isolation from everything else. Environmental forces will distort small pieces, keeping them from being exact copies of the whole."[42] In other words, at a purely physical level, the repetition *must* end as the magnification reaches a level in which the repetition is no longer structurally possible (the atomic level). At the same time, precisely because of the relations that exist between beings with one another and the effects they all have on one another, perfect repetitions will be impossible since the perfect repetition would entail total solitude and complete separation from all others. Accordingly, one can say that the statistical distribution, the distortion, and the uneven scaling of self-affine fractals is the result of its being *both* singular and plural. This is well illustrated by the example of a mountain given by Frame and Urry. As they tell us, "[o]ften mountain ranges scale differently in vertical and horizontal directions. Geological forces, about the same along the length of the mountain range, push up the rocks with one scaling factor; meteorological forces weather down the rocks with a different scaling factor. Different horizontal and vertical scalings make these fractals self-affine."[43] Here we can see with clarity that the scaling distortions are themselves the result of an interaction. The mountain's exposure to weather means that its scaling cannot be linear in all directions.

We can see here how scaling differences relate to a statistical repetition. What

[42] Frame and Urry (2016, p. 56)
[43] Frame and Urry (2016, p. 10)

makes these natural formations fractal is that they are iterative and repeating structures, however, what makes them self-affine rather than self-similar is that this repetition, while close enough to be recognized as a repetition, is far enough from itself to be unpredictable *as a precise pattern*. Accordingly, what we are left with is the knowledge that a repetition that is *related* to the previous repetition will occur at certain scale levels, but the exact values are unknown to us because the scaling factors differ from one another in different directions. In other words, through these interactions that natural beings undergo, scaling factors shift around in such a way that the next iteration cannot be predicted except as a possible range based on a guess about how it is relating to that which surrounds it. This is why a self-affine fractal, besides being "statistically" self-similar as Kenkel and Walker describe it, can also be defined quite simply as fractals "characterized by different scaling factors in different directions"[44] which we saw very clearly in the example of the mountain.

While the difference between having the same scaling factor in all directions versus having different scaling factors in different directions might not seem like a lot at first glance, its implications for calculation are significant. Since the fractal dimension is calculated by means of this scaling factor, such a variance complicates it to such an extent that Frame and Urry can tell us that "[f]or fractals that are self-affine…finding the [fractal] dimension is much, much more difficult, and *no one knows how to do this in general.*"[45]

We can see this difficulty clearly illustrated, for instance, in a description that Mandelbrot gives us of a scalar Wiener Brownian record. As he tells us, unlike a self-similar fractal where "the divider exponent D coincides with all other forms of the fractal dimension, e.g., the similarity, box, or mass dimension," for a self-affine curve "a full description in terms of the fractal dimension is complex. Each dimension splits into a *local* and a *global* value, separated by a crossover. Globally, all the basic methods of evaluating the global fractal dimension…yield 1; that is, a self-affine fractal behaves globally as if it were not a fractal. Locally, the box and mass dimensions are 1.5, but the divider dimension is $D=2$."[46] In other words, in contrast to a self-similar fractal like a Sierpinski gasket where we can say that

[44] Frame and Urry (2016, p. 6)
[45] Frame and Urry (2016, p. 162; emphasis mine)
[46] Mandelbrot (1985, p. 257)

its fractal dimension is approximately 1.585 and this figure is the same both locally and globally and across all forms of the fractal dimension, with a self-affine fractal, this all breaks down. As Mandelbrot notes, interestingly, a self-affine fractal does not appear fractal globally even though it breaks down into a fractal structure locally, however, this local fractal structure itself varies according to how it is being measured.

What we see here is another form of exceedance. If the self-similar fractal can be thought of according to its dimensional non-coincidence, it nevertheless retained a constant scaling factor and, therefore, a predictable iterative pattern that allowed its fractal dimension to be calculated. In the self-affine fractal, however, we approach the incalculable. Its complexity now goes beyond dimensional discordance and towards another form of irreducibility. That is, if the self-similar fractal's dimensional discordance can be thought of along the lines of an irreducible roughness, the self-affine fractal's incalculability can be thought along the lines of the unpredictability and irreducibility of interaction, exposure, and being-with. Through its exposure to other beings, the self-affine fractal cannot achieve perfect self-similarity such that this difficult or impossible calculation of the fractal dimension can be seen as a marker of its exposition to what is outside itself and its susceptibility to others. If the mountain were not in a world exposed to weather, then its fractal dimension would be easier to calculate but then, of course, it would not be a mountain.

This is where we can see how the self-affine fractal becomes a better model for Nancy's ontological picture. Unlike the self-similar fractal, a self-affine fractal is a singularity produced as a transitory cohering structure susceptible to, in contact with, and determined by its relations to others in its exposure to them. The self-affine fractal becomes itself through this touching of surfaces which ensures that its iterations are never *identical* from one moment to the next, even if they may be *close* to one another. Accordingly, the self-affine fractal's internal structure is already a being singular plural. In its non-identical iterative structure, we have a circulation of beings which are *with* one another and *close* to one another, but not the same as one another and never quite crossing over one another although related to and in touch with one another. At the same time, this internal structure of the self-affine fractal is itself the result of its being exposed to and in touch with other beings in the spacing out of the world. The self-affine

fractal thus improves upon the overly constricting structure of the self-similar fractal by introducing, beyond dimensional non-coincidence, an internal non-coincidence since the copies of itself that the self-affine fractal is composed of are, nevertheless, not identical to it or to each other. And this corresponds to its exposure to other beings and thus to its transitory nature. That is, the self-affine fractal, because of its exposure to others, is subject to being affected and changed by them and, therefore, potentially disappearing or morphing into a new structure, a new circulation, a new community of beings. As we enter the world, we thus find that self-affinity must be the case rather than self-similarity precisely because the world is *shared* and is that space in which that sharing occurs. The incalculability and self-differencing at the heart of the self-affine fractal is the result of its being in a world *with* others and creates its own kind of infinity.

Here, we can once again note that this picture is also an ecological one. To the extent that ecology is understood as both "the interrelationship between any system and its environment" as well as "the product of this",[47] we find here also an implied fractality, relationality, and circulation. That is, an ecology is both the relations between beings *as well as* the product of the relation between these beings which may stabilize itself as something discrete and transitory. In order to specify further how it is that these fractal singularities form and produce themselves as discrete and transitory beings within a context of generalized circulation, we will look at some features of Brownian motion and how this affects and produces aggregation.

CIRCULATION, AGGREGATION, INCLINATION, AND BROWNIAN MOTION

Thus far we have discussed coastlines and mountains, that is, natural formations at an already relatively large scale level. But what happens at the other end of the spectrum? In an effort to clarify further how it is that singularities form themselves as discrete and transitory circulations of beings, it will be useful to go down to the particle level. In describing the different models for planetary formation, Frame and Urry tell us that:

> One approach…is called *ballistic aggregation* (BA). Here, particles travel in straight

[47] Ecology, N." Oxford English Dictionary, Oxford UP, September 2024.

> lines (ballistic trajectories) until they collide and then stick together…form[ing] dense, compact clusters…This growth suffers from a problem. To move in about straight lines through the nebula, particles must be fairly large and moving relatively quickly. Under these circumstances, collisions tend to fracture clusters, not add to them.[48]

In other words, the smooth model of straight line collisions fails to explain how aggregation could begin. It can only work with the assumption that particles are already fairly large, but if particles are already fairly large then how did they aggregate if they could not aggregate according to the straight line model of ballistic aggregation? This is where a different, fractal model must be introduced instead. As Frame and Urry continue:

> At least for the growth of small clusters, a better model is *diffusion-limited aggregation* (DLA)…This model begins with micrometer-size particles performing a random walk (Brownian motion, Sect. A.10) caused by thermal impacts of other particles. These random walkers collide and stick, growing branches, and side branches, and side branches off side branches, and so on. Most growth is around the periphery of the cluster because the outer branches screen the inner from particles which randomly wander in…form[ing] wispy fractal clusters…As these clusters grow, they are less likely to stagger into one another…and the clusters begin to zip along in straight lines until they collide with each other and stick…[in] *ballistic cluster-cluster aggregation* (BCCA).[49]

What we see here is that *before* particles can move in straight lines to aggregate at a higher scale level, they must first be moving in "random" directions at a lower scale level where such "random" movements form the condition for their aggregation into wispy clusters. Of course, these "random" movements are not *entirely* random, they can be modelled fractally by Brownian motion. As Perrin (1909) describes it for us:

> In a fluid mass in equilibrium, such as water in a glass, all the parts appear completely motionless. If we put into it an object of greater density, it falls…However, it would be difficult to examine for long a preparation of very fine particles in a liquid without observing a perfectly irregular motion.[50]

This "perfectly irregular motion" was first noticed by Scottish botanist, Robert Brown—from which it receives its name—in relation to the movement of

[48] Frame and Urry (2016, p. 80)
[49] Frame and Urry (2016, p. 81)
[50] Cited in Mandelbrot (1983, p. 12)

pollen particles. What this clarifies, however, is that when a multiplicity of small particles is together with one another in a circulation, they produce fractal movement paths. This is why Frame and Urry tell us that Brownian motion is "a good model of the collective behavior of independent agents with identical distributions."[51] And their very word phrasing is telling since they are not referring either just to independent agents (singularities) or to collective behavior (plurality) but to the "collective behavior of independent agents" (singular plural). Accordingly, we can see that at this very small scale level of particles being with one another, we already find a circulation wherein no priority can be set between the singular and the plural but, rather, what occurs is produced by their conjunction. Brownian motion produces fractal movement paths precisely because of the exposition of each particle to all the other particles and, as we saw, these movement paths may produce collisions that can aggregate the particles into larger clusters (or singularities) which, themselves, becomes new sites of exposure capable of aggregating into yet larger clusters (or singularities) and so on and so forth.

But what is it that drives this or that aggregation? What is interesting about Brownian motion is that it is itself already plural. That is, Brownian motion comes in a few different types. As Frame and Urry tell us, not all Brownian motion appears to be statistically random, "some processes remember their past actions. Fractional Brownian motion adds memory to Brownian motion."[52] The way in which this memory of past action affects Brownian motion divides itself into two types. As they explain further, "[f]or standard Brownian motion ($\alpha = 1/2$) the present is unaffected by the past. For persistent ($\alpha > 1/2$) and anti-persistent ($\alpha < 1/2$) fBm the past does influence the present, but in opposite ways. For persistent we remember what we've just done and we like it, so let's do it again…For anti-persistent we remember what we've just done and we don't like it, so let's do the opposite."[53] And this will correlate with roughness, that is, anti-persistent Brownian motion will be relatively rougher than persistent Brownian motion, with standard Brownian motion finding itself between them.

So, it would seem that within these different types of Brownian motion, we

[51] Frame and Urry (2016, p. 284)
[52] Frame and Urry (2016, p. 284)
[53] Frame and Urry (2016, pp. 285-286)

can detect, at least in some cases, a certain inclination in one direction or another. The particle might tend to repeat what it has done in a way that we might call "stickier" than standard Brownian motion, or the particle might tend to avoid what it has done. To the extent that the particle seeks to repeat what it has already done, it makes its path smoother and to the extent that it avoids its past, it becomes rougher and more complex. Particle movement thus begins to appear to be something which may be attracted or inclined in one direction or another although, as we have noted, even the particle that would seek to smooth itself out will never achieve smoothness since it has already found itself in an inescapable exceedance unless, of course, it simply stops moving and becomes a point, however, then it would no longer be a curve or a motion path but something else.

All this approaches what Nancy calls the *clinamen*. That is, the circulation of Being in the opening of space occurs because things start moving and this movement begins as a *clinamen*, a bending towards this way or that. It is by means of this *clinamen* that things happen, beings move, and meaning is produced by way of this exposition and this circulation in an unpredictable fashion that can never close down precisely because the *clinamen* of each is confronted by the *clinamen* of the other in a space in which none can dominate or master the others even if these different *clinamina* may, nevertheless, come together and aggregate into new singularities with their own *clinamina*. As he says, "one cannot make a world with simple atoms. There has to be a *clinamen*. There has to be an inclination or an inclining from one toward the other."[54] If the world were *just* made of atoms alone, there would be no world; there would just be a stasis of motionless atoms. The world *is,* as it is, precisely because these atoms *move* and, in this movement, make something happen by coming into non-fusional contact with one another. Hence, Nancy clarifies further that "[s]ingularity never takes place at the level of atoms, those identifiable if not identical identities; rather it takes place at the level of the *clinamen*."[55] Here, we can see that the implication is that the *clinamen* and the spacing out of Being are to be related to one another. That is, if the singularity does not take place at the level of atoms but at the level of the *clinamen*, it is because it is through the *clinamen* that the singularity begins to circulate in the open space

[54] Nancy (1991a, p. 3)
[55] Nancy (1991a, p. 6)

in which it will be exposed to all the rest (and their *clinamina*) and, in this way, produce its meaning and relate to others in the production of other singularities and other meanings.

In the foregoing, I have sought to show how, first of all, fractals appear increasingly to be a general structure of the world as we find them in the vast array of self-affine natural formations ranging from, as we saw, micrometer size particle clustering to planet formation, coastal extent to mountain weathering but, second of all, I have also sought to show how this self-affine fractal structure itself embodies important features of Nancy's view of ontology. Accordingly, we might say that what fractal geometry shows us—or exposes for us—as we think it with Nancy, is (rewriting the title of one of Mandelbrot's books) a *fractal ontology of nature*.

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