

## MATHEMATICAL FRAMEWORKS FOR CONSCIOUSNESS

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**ABSTRACT:** If Awareness is fundamental in the universe, mathematical frameworks are better suited to reveal its fundamental aspects than physical models. Awareness operates through three fundamental laws which apply at all levels of reality and is characterized by three universal powers. We explore and summarize in general terms mathematical formalisms that may take us as close as possible to conscious awareness, beginning with the primary relationships between the observer with the observed, using a Hilbert space approach. We also examine insights from category theory, and the calculus of indications or laws of forms. Mathematical frameworks as fundamental languages of our interaction with the universe should be further developed with consciousness being the driving force.

**KEYWORDS:** Mathematical formalism; Hilbert space; Category theory; Laws of forms; Observer; Observed; Qualia

### INTRODUCTION

We explore below mathematical formalisms that we believe are useful to formally approach conscious awareness. Mathematical approaches are better suited than physical theories as the latter are developed for an external world, separated from conscious awareness. Revising quantum mechanics to accommodate certain consciousness-based phenomena is fraught with new assumptions, such as revising the Born rule. Mathematical frameworks, on the other hand, as fundamental languages of our interaction with the universe, are more naturally suited. Here we begin with a review of the primary relationships between the observer with the observed, using a

Hilbert space approach, suitable for primary qualia<sup>1</sup>. Then we examine insights from category theory, and the calculus of indications or laws of forms.

### THREE LAWS OF NATURE AND QUALIA

In previous works<sup>2 3</sup> we have developed generalized Complementarity as a foundational Law for all realms of reality. The fundamental relationships between subjects and objects form the foundation of qualia. The world of experiences reveals three fundamental Laws of Nature, reflected in quantum mechanics (QM) but even beyond to psychophysical, mental, in fact in all human endeavors, *and* in the way Consciousness objectifies the world: *Complementarity, recursion, and creative interactivity*.

*Complementarity* (or *Integrated Polarity*) is the first unifying Law, where ultimately the *apparent opposites become unified at the deeper levels* of universal Consciousness. Complementarity is at the foundation of the Copenhagen Interpretation (CI) and the von Neumann Orthodox QM interpretation. As complementary relations are too ubiquitous, implying that QM is the starting point for developing a scientific framework of consciousness, but as we have argued, more naturally developed in mathematical approaches.

The second Law is *Recursion* (or *Correspondence*), which can be simply stated, “as here, so elsewhere”, “as above, so below”<sup>4</sup>. Recursion allows science to be conducted as a universality of scientific descriptions is assumed, with all physics laws applying everywhere.

The third Law, *Creative Interactivity*, provides a framework of interactions at many different levels, such as interactions between subjects and objects, between sentient beings (in which case it takes on the special form of *Sentience*<sup>3</sup>); between stars and planets, cells and cells, etc.

The three Laws *give meaning to the universe*, they are the principles of organization and manifestation of the cosmos. Along with the three Laws, Awareness projects the

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1 Kafatos, M.C. (2015). “Fundamental Mathematics of Consciousness”, *Cosmos and History: The Journal of Natural and Social Philosophy*, 11(2): 175-188 <http://www.cosmosandhistory.org/index.php/journal>.

2 Kafatos, M., Nadeau, R. (1990; 2000). *The Conscious Universe*, Springer Verlag, New York. And, Kafatos, M. (2011). “The Science of Wholeness”, in *Analecta Husserliana*, T. Tymieniecka, A. Grandpierre (edit.), Springer Science, Business Media, B.V.

3 Theise, N.D., Kafatos, M.C. (2016). Fundamental Awareness: A Framework for Integrating Science, Philosophy and Metaphysics, *Communicative & Integrative Biology* 9(3): 00-00.

4 Theise, N. D., Kafatos, M. C. (2013). “Sentience Everywhere: Complexity Theory, Panpsychism & the Role of Sentience in Self-Organization of the Universe”, *Journal of Consciousness Exploration & Research*, 4, (4): 378-390.

cosmos through a very large, if not infinite, number of powers. Three are universal and most important: Will, Knowledge and Action<sup>5</sup>.

We next focus on *qualia*, individual instances of subjective, conscious experience. The so-called “hard problem”<sup>5</sup> addresses the seemingly impossible task for science to account for experience in terms of physical theories. Experience cannot be taken out of a quantum-based ontology, as observation implies experience, see for example von Neumann<sup>6</sup>. The issue of what is the meaning of collapse and the role of observation has recently been addressed<sup>7</sup>: Quantum non-local, eraser experiments actually imply the existence of a universal Observer (O) beyond space-time. It is *structured or organized information* that is responsible for the collapse of the wave function and not some mysterious mental action tied to observation by a human observer<sup>6 8</sup> (o). This actually fits well in the mathematical framework described below.

#### MATHEMATICS OF PRIMARY SUBJECT OBJECT RELATIONSHIPS

The mathematics has been developed and presented elsewhere<sup>1</sup>. The view proposed, on which we build here, is that working with physical theories alone will not lead to a framework of consciousness and such efforts are doomed to fail. The starting point, is an *ontological assumption, an axiomatic* approach similar to all mathematical formalisms: Here, *universal Awareness operates at every level of reality*. The three Laws, allow universal Awareness, which otherwise would be unmanifest and unknowable, to operate and give rise to all subjective experiences. Qualia, give rise to all levels of subjective experience and are the fundamental building blocks of the Conscious Universe. In the mathematical formalism,

$\langle \rangle$  is the symbol for absolute undifferentiated Consciousness

$\rangle$  is the symbol for Subject, i.e.  $\rangle = \text{“I”}$

$\langle$  is the symbol is for Object, i.e.  $\langle = \text{“That”}$

5 Chalmers, D.J. (1995). "Facing Up to the Problem of Consciousness", *Journal of Consciousness Studies* 2 (3): 200- 219.

6 von Neumann, J. (1955). *Mathematical Foundations of Quantum Mechanics*, translated by Robert T. Beyer, Princeton University Press, Princeton, NJ.

7 Narasimhan, A., Kafatos, M.C. (2016). “Wave Particle Duality, the Observer and Retrocausality”, in AIP Retrocausality Conference, D. Sheehan (edit.), <http://arxiv.org/abs/1608.06722>.

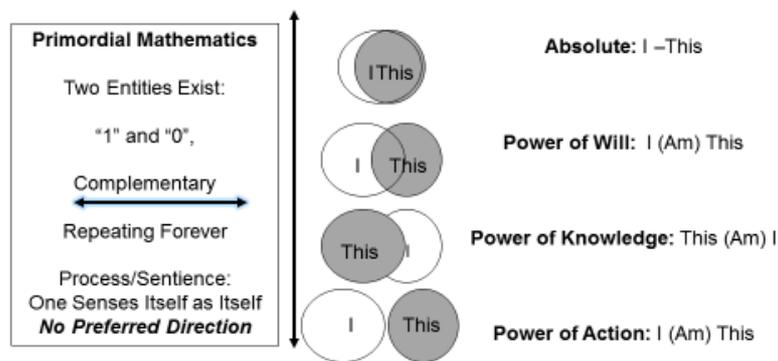
8 Stapp, H.P. (2009). *Mind, Matter and Quantum Mechanics*, Springer Verlag, Berlin, Heidelberg.

Moreover, the  $A$  symbol is “Am”. We then use the convention  $A| \rangle = | \rangle$  “I Am”.  
 While  $\langle | A^+ = \langle |$

means “That Am I”. “I Am” is existence of Subject; while “That Am” is existence of Object. Subject and Object are *not* yet differentiated, they form the primordial qualia relationship between what eventually will become a separated object from subject, the *fundamental complementarity*. The adjoint  $A^+$  is used in keeping with Hilbert space formalism. Further details can be found in Kafatos<sup>1</sup>. There are *five universal and logical (mathematical) statements*:

- I
- That
- I (Am) That
- That (Am) I
- I (Am) That

**Structure-less, Infinite, Subject/Object are One**  
 Separation of Subject from Object is Potential, *not* Actualized



**Figure 1:** Subject-Object relationships where there is no separation. The universal powers of Consciousness are indicated.

All three fundamental Laws, complementarity, recursion and creative interactivity, are operating at these five levels (see Fig. 1): For example, Complementarity is operating as the fundamental relationship between Subject and Object, etc. However, as we move next to the level of breakdown of the above universal relationships, “persistence” of what is the Power of pure *Will* gives rise to (limited) power of will to

know and act, etc. When the Subject and Object become separated, a certain *veiling of Consciousness occurs, which may be manifest in quantum non-locality*<sup>9</sup>. The simple logical statements presented above break down, the universal  $| \rangle$  and  $\langle |$  (equivalent to two vector states in Hilbert space) now become limited subjects and limited, the mathematics becomes much more complex, instead of **o** and **I**, we now have an infinite set of vectors, representing infinitely many sentient beings, interacting with each other and sensing objects. What occurs is the logical statement **I** (Am) **Not** that, or **That Not** (Am) **I**. Here the symmetry that applied to the first five levels breaks down, which may *allow space, time and mass to arise*; as well as limited will (of subjects), limited knowledge (of objects), and limited action (between subjects and objects), *providing an account of qualia*.

The world appears as classical, composed of separate subjects and objects. However, the general principles of complementarity, recursion, and sentience still hold but now in an infinitely complex set of entities. The universe is then born. In summary, Consciousness or Universal Awareness produces object-subject relationships. *Fundamental mathematics at the first five “pure” levels is the expression of the fundamental Laws*. Science, through mathematical physics involving dynamics (springing out of fundamental mathematics of the three Laws) is utilized by our minds and intellect to provide the qualia of understanding of our environment.

Finally, when the Subject – Object identity breaks down, we also have the universal Observer (Subject) that *appears* to be separate from the individual observer (o), in space-time. It is remarkable that quantum eraser experiments actually reveal this separation.

#### MATHEMATICS OF CATEGORY THEORY

The next formalism that is particularly useful to qualitative descriptions of fundamental Consciousness and the Laws of Nature is Category theory. Here we follow closely the development by Struppa *et al.*<sup>10</sup>:

Category theory was developed as a general framework for many fundamental concepts like sheaves, presheaves etc. in algebraic analysis. Initially sheaves, presheaves and functors were thought as purely mathematical objects. Now with

9 Kafatos, M.C., Kak, S.C. (2015). “Veiled Non locality and Cosmic Censorship”, *Physics Essays*, **28**: 182-187 <http://physicsessays.org/browse-journal-2/product/1242-9-menas-c-kafatos-and-subhash-kak-veiled-nonlocality-cosmic-censorship-and-local-observations.html> <http://arxiv.org/abs/1401.2180>.

10 Struppa, D. C., Kafatos, M., Roy, S., Kato, G., and Amoroso, R. L. (2002). “Category Theory as the Language of Consciousness”, *Noetic Journal* **8**(3): 271-281. And references therein.

the advancement of quantum cosmology it is believed that these abstract mathematical objects may play a significant role in physics... Here, a relationship between the flow of quantum information and the causal structure of a quantum space-time is discussed. The concept of elementary screens is defined here as set of events at which the observables of a holographic cosmological theory may be measured. These screens are spacelike-2 surfaces on which relevant degrees of freedom of the theory exist. Then they define a class of quantum space-times consisting of causal networks of such screens. This is known as screen networks. Also, in a partially ordered set of screens, in which two screens are related,  $s \leq t$ , when one of the future components of  $s$  precedes one of the past components  $t$ . There can be at most one edge (covering relation) from  $s$  to  $t$ . This means that, if  $s$  is in the immediate past of  $t$ ,  $t$  can see one side of  $s$ .

Then one can turn a screen network into a network of elementary quantum mechanical systems. In doing so it is assumed that quantum information propagates without change between screens and undergoes non-trivial evolution only when going through a screen. This can be done by assigning a Hilbert space to every edge of the screen network, and two (unitary) evolution operators to each screen. Then we can define the *edge screen network* ( $ES$ ) as the partially ordered set whose elements are edge-sets, sets of space-like separated edges  $a, b, \dots$  in the screen network  $S$ .

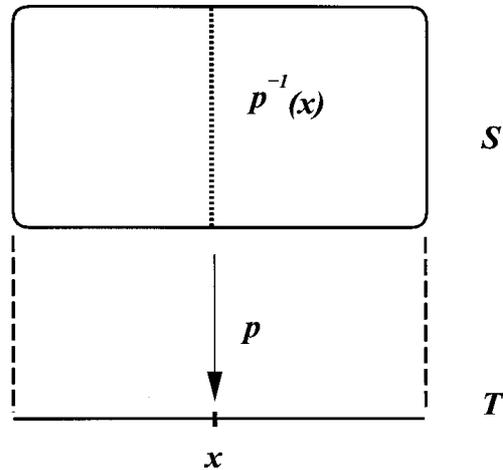
A *quantum screen network* ( $QS$ ) can be defined as a *Functor* from the edge screen network  $ES$  to the category of Hilbert spaces.

Hence,  $QS$  is the *Functor*

$$QS : ES \rightarrow \text{Hilb.}$$

Such that for every edge-set  $a$  in  $ES$  there is a finite-dimensional Hilbert space  $H(a)$  in  $QS$ .

Based on these concepts a discrete background independent holographic theory can be formulated where it is possible to build up a model of transmission of quantum information. In fact, an area of screen is defined as a measure of its capacity as a channel of flow of quantum information. However, this framework is its infancy and needs to be further developed. This may turn out to be a special case of our framework developed here.”



**Figure 2:** Given a topological space  $T$ : A sheaf on  $T$  is a triple  $(S, T, p)$  where  $S$  is another topological space,  $p: S \rightarrow T$  is a continuous map, such that for any  $x$  in  $T$ , the fiber  $p^{-1}(x)$  is a topological Abelian group, with the topology induced by  $S$ . Sheaf can be generalized beyond groups.  $p^{-1}(x)$  is a “stalk” for the sheaf  $S$ .

Various QM phenomena, which actually form subsets of the three Laws of Nature, can be discussed in the context of category theory, within universal Consciousness, as they apply to the physical world. Here we give a few examples:

*Complementarity:* Niels Bohr<sup>11</sup> proposed the principle of complementarity in QM as a logical framework for the interactions of observers with nature, in the conscious construction of reality. Bohr maintained that the most fundamental complementarity is the relationship between object and subject. In QM, complementarity manifests in the wave/particle duality in the universe. If conscious entities are described as *presheaves* on  $T$  (where  $T$  is a general topological space), their *complementarity* nature is embedded in the very way in which presheaves and sheaves are constructed. Or, when sheaves are looked upon from the point of view of the triple  $(S, T, p)$ <sup>12</sup>, they exhibit a localized nature, while when they are described through the notion of complete presheaves, they assume a diffuse character. Interestingly enough, while both descriptions are adequate ones, on each given theorem, only one or the other

11 Bohr, N. (1934). *Atomic Theory and the Description of Nature*, Cambridge University Press, Cambridge. And, Bohr, N. (1958). *Atomic Physics and Human Knowledge*, Wiley, New York.

12 This is beyond the scope of this work but for the mathematically interested: A sheaf on  $T$  is a triple  $(S, T, p)$  where  $S$  is another topological space, and  $p: S \rightarrow T$  is a continuous map such that for any  $x$  in  $T$ , the fiber  $p^{-1}(x)$  is a topological abelian group, with the topology induced by  $S$  on it.

representation is used in practice, in accordance with the principle of complementarity.

*Recursion* leads to structures and self-organization, embedded in scientific descriptions of the universe. The structures forming from recursive relationships are self-organized at several different levels. At the lowest level one has *elements*, which come together to form *groups* or other algebraic structures. Furthermore, these algebraic structures come together to form *categories*, and beyond. Consciousness is responsible for self-organization and greater complexity at global levels. The arrow of forming more complex structures can be understood in category theory as “generalized time”.

The QM phenomenon of *non-locality* arises as a typical property of an element of a sheaf (or a presheaf). Since  $T$  is generalized time (in physics known as space-time), an element of a sheaf is globally defined on such a space-time, but its local expressions are linked by properties which are non-local.

What has been presented here can be taken to the next level through the mathematical system of *temporal topos*<sup>13</sup> of sheaf co-homology.

#### MATHEMATICS OF LAWS OF FORM

The last mathematical framework we examine is the *Laws of Form* developed by G. Spenser-Brown<sup>14</sup>. In his words:

One of the motives prompting the furtherance of the present work was the hope of bringing together the investigations of the inner structure of our knowledge of the universe, as expressed in the mathematical sciences, and the investigations of its outer structure, as expressed in the physical sciences...our common experience of perception, the inside world, can be revealed by an extended study of the outside world, then an extended study of this inside world will reveal facts first met with the world outside: for what we approach, in either case, from one side or the other, is the common boundary between them.”

We clearly have here a fundamental complementarity as realized by Spenser-Brown himself. As in Boolean algebras, Spenser-Brown developed primitive equations for his *calculus of indications*, utilizing definitions, axioms, theorems, etc. His primary arithmetic includes the following expressions, given in graphical form. Following the work of R. Pizzi<sup>15</sup>, we present here some of the main expressions for the *indications*:

<sup>13</sup> Kato, G. (2013). *Elements of Temporal Topos*, Abramis, Suffolk, UK.

<sup>14</sup> Spenser-Brown, G. (1969). *The laws of Form*, Allen & Unwin, London.

<sup>15</sup> Pizzi, R. (2016) “Consciousness as Emergent Property of the Interaction between Cognitive Levels”, in *Consciousness: Social Perspectives, Psychological Approaches and Current Research*, L. Alvarado (edit.), Chapter 10: 205-210, Nova Science Publishers.

Distinction (where any distinction is *indication of a value*)



- And Unmarked (*unmarked state or non-indication*) Blank

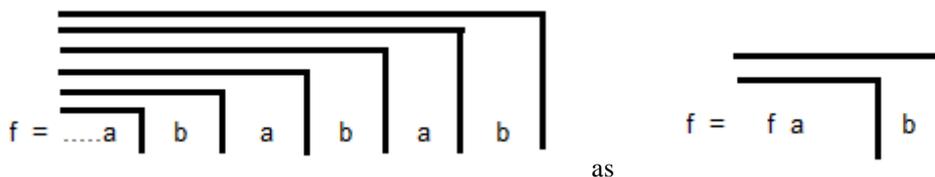
- Condensation



- Cancellation



- This calculus of indications is handling self-reference and making it consistent, by expressing extended forms like:



Where the expression *f re-enters* into itself, or *in-forms itself* as in Moebius ring.

Extending Spenser-Brown, Maturana and Valera<sup>16</sup> introduced a third *autonomous* state<sup>14</sup>:



The basic idea is that self-reference is now included in the calculus, by introducing a third state which forms itself in an autonomous way by self-indication. The third state is *not reducible* to the laws of the two early states, but comes out from the co-operation of the elements of a two-valued systems.

According to B. Russell,

...Spencer Brown has revealed a new calculus of great power and simplicity.

<sup>16</sup> Maturana, H., Varela, F. (1973). *De Máquinas y Seres Vivos: Una teoría sobre la organización biológica*, Editorial Universitaria, Santiago.

The calculus of indications and its extensions may well be a mathematical language appropriate for addressing consciousness, expressed in well-defined graphical equations.

## CONCLUSIONS

Building upon the quantum framework, we realize today that quantum theory has many profound implications for building a mathematical framework of the nature of consciousness. As such, what used to be in the domain of philosophy and metaphysics, can now be approached by science, utilizing the language of science, mathematics.

We explored and summarized in general terms mathematical formalisms that may take us as close as possible to conscious Awareness, beginning with the primary relationships between the observer with the observed, using a Hilbert space approach. We also examined insights from category theory, and the calculus of indications or laws of forms. Mathematical frameworks as fundamental languages of our interaction with the universe should be further developed, keeping in mind that the driving force is exploration of consciousness. We have explored three mathematical formalisms that we believe are particularly useful to formally approach conscious Awareness. We emphasize that mathematical approaches are better suited than physical theories, as the latter are developed for an external world, separated from conscious Awareness.

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