NUMBERS: HARMONIC RATIOS AND BEAUTY IN AUGUSTINIAN MUSICAL COSMOLOGY

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ABSTRACT: This article explains how, according to the Pythagorean concept of the first four numbers (1-2-3-4), Augustine develops his own musical cosmology in Christian theological context. Augustine agrees with the Pythagorean cosmology that numbers as the essence of the cosmos have been measured and arranged in a harmonic order and the sequence of the first four numbers perfectly illustrates the harmonic order. Based on the unchangeable nature of numbers, Augustine argues that the ratios of the first four numbers (1/1, 1/2, 2/3, 3/4) are not only the fundamental of harmonic law, but also the highest *Modus*, by which God created all beauties/forms.

KEYWORDS: Numbers; Musical cosmology; Music; Beauty/form; *Modus*; Harmony; Goodness

INTRODUCTION

Augustine's interest in numbers pervades many of his writings. The core issues of his early writings were especially shaped by the concepts of numbers borrowed from the Pythagoreans, but he was successful in harmonizing these with his Christian faith. In *De Musica*, one of his early writings, Augustine specifically discusses the nature of numbers and the unchangeable numerical principles of music theory. In this apologetical treatise, he adopts the Pythagorean musical cosmology in Christian theological context in order to argue that all things were created and measured by God according to the harmonic order. Although number is not the most popular topic among Augustinian scholars, it has gained some attention. In her short article "Measure, Number and Weight in Saint Augustine's Aesthetics," Carol Harrison, without discussing the issue at length, mentions that the terms number, beauty and

form, "are used almost synonymously by Augustine, in reference to the Son, the second person of the Trinity."[1] J. Roland E. Ramirez's short article "Augustine's Numbering Numbers" addresses the relationship between the power of numbering and the immortal soul. Ramirez provides an important insight: "It is, however, only because the human soul is incorporeal and immortal that the incorruptible, eternal numbering numbers can be naturally discovered to have a residence in the human soul."[2] Catherine Pickstock's "Ascending Numbers: Augustine's *De Musica* and the Western Tradition"[3] digresses from the subject of Augustine's music theory, while the word "numbers" in the title is more or less symbolic, as the article addresses neither the topic of number nor the numerical ratios of music theory. Montague Brown's "Augustine on Beauty, Number, and Form" discusses the intrinsic relationship between numbers, beauty and form, and he presents a convincing reading, "Number is spoken of as God's knowledge of and very presence to created things, [which] he has created according to number."[4] Ellen Scully discusses the Pythagorean numerology in her "*De Musica* as the Guide to Understanding Augustine's Trinitarian Numerology in the *De Trinitate*." Although she does not address the ratio of musical harmony, she explains the theological analogy of numbers within the Trinity. Further, she points out the significance of numbers in Augustine's metaphysical argumentations. As she notes,

> He is enough of a Pythagorean to understand the beginning of all numbers as the beginning of all things, but it is precisely the focus on numbers that allows him to pursue this question in the intelligible rather than material realm.[5]

Previous research only discusses the concept of numbers without analysing the harmonic ratio and addressing the beauty of God. This article explains how, according to the Pythagorean concept of numbers, Augustine demonstrates his musical cosmology and argues that the progression of the first four numbers (1-2-3-4) is the highest *Modus* of God that determines both physical and metaphysical beauty.

[1] Carol Harrison, "Measure, Number and Weight in Saint Augustine's Aesthetics", *Augustinianum*, vol. 28, no. 3, 1988, pp. 595-6.
[2] J. Roland E. Ramirez, "Augustine's Numbering Numbers", *Augustinian Studies*, vol. 21, 1990, p. 160.
[3] Catherine Pickstock, "Ascending Numbers: Augustine's *De Musica* and the Western Tradition", in Lewis Ayres and Gareth Jones (eds.), *Christian Origins: Theology, Rhetoric and Community*, London, Routledge Published, 1998, pp. 185-215.
[4] Montague Brown, "Augustine on Beauty, Number, and Form", in F. Yong, M. Edwards and P. Parvis (eds.), *Studia Patristica*, vol. 43, Leuven, Peeters, 2006, p. 34.
[5] Ellen R. Scully, "De Musica as the Guide to Understanding Augustine's Trinitarian Numerology in the De Trinitate", *Augustinian Studies*, vol. 44, no.1, 2013, p. 99.

THE NATURE OF NUMBERS

The numerical truth, according to the Pythagorean tradition, includes four subjects: arithmetic, music, astronomy and geometry. These four subjects are sister-disciplines under one subject, mathematics. Among them, only music is the combination of the principles of both natural science and morality. When applying the unchangeable mathematic truth to searching the morality and the meaning of the world, neither the Pythagoreans, nor Augustine can avoid studying music which is measured numbers moving in the temporal realm. Since the truth of numbers is unchangeable and common to everyone, Augustine holds that to ignore the knowledge of numbers and music would be a great loss for Christians in their pursuit of the knowledge of God. He claims: "Ignorance of numbers prevents us from understanding things that are set down in Scripture in a figurative and mystical way…. An ignorance of certain elements of music also conceals many other things."[6] He insists that "nothing is surer than number, or more orderly than the recitation and measurement of feet… whatever is expressed in the nowise deceptive ratio of numbers is capable of delighting the ear and dominating rhythm."[7] When human ears enjoy a harmonic sound, in fact, they "are enjoying a certain measuring out of numbers in the sound."[8]

In *De Musica* I, Augustine analyses the nature of numbers, thus the topic of this book is *Quid sit musica eiusque motus et numeri* (The nature of music and its motion and number). Augustine explains that the finite numbers that can be perceived by the senses and analysed by reason are the footprints of the infinite numbers; then, by tracing the footprints, the secret sanctuary of infinity can inerrably be reached.[9] Numbers start from one, but go to endless infinity. Every number is individual and finite, but the union of all numbers is the incomprehensible infinity, which only belongs to God. In *De Libero Arbitrio*, Augustine explores the immutable and immanent nature of numbers and analyses the relationship between numbers and wisdom, raising several questions that address the relationship between the two: "Since the Holy Scripture put wisdom and number together, I would like to know whether these two things are contained in one category, or does one depend on the other, or is the one included in the other. Does number stem from wisdom, or is it in wisdom?"[10] Augustine regards the order of numbers and wisdom as the same thing. In order to illustrate his opinion,

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[6] Augustine, *De Doctrina Christiana*, 2.16.25, Patrologia Latina 34, J. P. Migne (ed.), Paris, 1865. (Note: In this article, English translations of Augustine's original texts are my own, unless otherwise noted.)

[7] Augustine, *De Musica*, 3.7.16, Patrologia Latina 32, J. P. Migne (ed.), Paris, 1841.

[8] Ibid., 2.2.2.

[9] Ibid., 1.13.28.

[10] Augustine, *De Libero Arbitrio*, 2.11.30, Patrologia Latina 32, J. P. Migne (ed.), Paris, 1841.

[10] Ibid., 2.2.2.

he quotes a biblical verse: "She [wisdom] reaches mightily from one end all the way to the other, and she [wisdom] orders all things sweetly."[11] He divides this verse into two parts to indicate numbers and wisdom respectively: "The power that 'reaches from end to end powerfully' is perhaps called 'number'; while the power which 'disposes all things sweetly' is now properly thought of as wisdom."[12] God, the Wisdom who is the source of all numbers, gives numbers to all things, but only grants man the power of reasoning to know wisdom, the unchangeable truth.

> All objects, even the least ones, have their own numbers; because wisdom did not grant, either to corporeal objects or to all spirits, the power to know, but granted it only to rational beings, as if it made in them a home for itself, from which it could arrange everything, including even the least object to which it has assigned number….[13]

Although human beings were endowed with the power of knowing, only few men have wisdom when their minds have ascended through the immutable power of numbers to the immutable truth. Augustine explains that, "[m]any people know numbers and can count numbers, but very few know wisdom." Further, it seemed to him that "wisdom is more venerable than numbers."[14] Augustine compares wisdom and number to the light and heat, which belong to one thing, fire, but have different dimensions and territories. Like heat, wisdom only affects objects which are close to it; while the number of light reaches objects far away.[15] But the one who diligently seeks wisdom, remains close to it through the unchangeable numeric truth, because "[w]isdom shows itself to the seeker in the guise of numbers embodied in all things of this world."[16] The guise of numbers is form, namely, beauty. Music is beautiful because of the power of numerical ratio, which is covered by temporal sound but can be recognized by human reason, namely, the power of the mind: "Mathematical ratios endure their splendour and serenity to be clouded by musical sound…because whatever the mind is able to see is always present and is acknowledged to be immortal, and numeric proportions seem to be of this nature."[17] Humans can know the unchangeable truth of number when their minds ascend to Wisdom, by which all numbers are numbered. The power of number is best illustrated by musical motion,

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[11] Wisdom 8:1

[12] Augustine, *De Libero Arbitrio*, 2.11.30.

[13] Ibid.

[14] Ibid.

[15] Ibid., 2.11.32.

[16] Ibid., 2.16.42.

[17] Augustine, *De Ordine,* 2.14.41, Patrologia Latina 32, J. P. Migne (ed.), Paris, 1841.

which is the temporal movement of pure measured numbers and floods into the human mind and guides it to Wisdom. Therefore, Augustine discovers that

> [a]mong all mutable things, which move because of the power of numbers, motions are most easily recognized in music, which furnish a path of rising to the higher secrets of truth. Along this path, Wisdom pleasantly reveals herself, and in every step of providence meets those who love her.[18]

The relationship between number/beauty and wisdom explained by Augustine can refer to the Pythagorean concept that regards the knowledge of numerical truth (astronomy, music, arithmetic, and geometry) as the fourfold way to wisdom. According to the Pythagorean doctrine, "Not only do all things possess numbers; but, in addition, all things are numbers."[19] Augustine echoes this doctrine, stating:

> Behold the sky, the earth, and the sea, and all that are in them or shine from above, or creep downwards, or fly, or swim, they all have forms because they have numbers: to take away these numbers, would mean that nothing would remain. So, they cannot exist unless their beings have these numbers.[20]

Augustine tirelessly seeks a logical foundation for his argumentation according to the inviolable nature of numbers. He recognizes that the power of right reasoning of the soul and the power of the numbers in the created realm are inherent in all knowledge. He explains that "nobody can aspire to a knowledge of those matters without this twofold science: the science of right reasoning and the power of numbers."[21] Right reasoning is the power of numbering and knowing things according to the way things should be known, while the power of numbers of the created realm is the principle which conditions everything's nature and *modus*. In short, the immortal science consists of reason (the inner number) and the numbers of the outside world. Since all things have been measured by God, reason is responsible for knowing things as they should be known according to the principles which condition their existence. "In all these branches of study, all things are presented to reason as numerically proportioned."[22] Numbers determines the essence of the corporeal world, which can only be deciphered by the power of reasoning, as Augustine posits: "Reason advanced to the province of the eyes. Scanning the earth and the heavens, it realized that nothing pleased it but beauty; and in beauty, form; and in form, dimensions; and in

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[18] Augustine, *Epistola 101*, ad Memorio, 3, Patrologia Latina 33, J. P. Migne (ed.), Paris, 1865.

[19] Thomas Heath, "Numbers and the Universe", in *A History of Greek Mathematics*, vol. 1, Oxford, Clarendon Press, 1921, p. 67.

[20] Augustine, *De Libero Arbitrio*, 2.16.42.

[21] Augustine, *De Ordine*, 2.18.47.

[22] Ibid., 2.15.43.

dimensions, number."[23]

In order to explain Christian faith approached from the science of numbers, Augustine frequently quotes this biblical verse: "You have ordered all things in size and number and weight."[24] He argues that the truth about the numerical nature of the world discovered by philosophers conforms to the revelation recorded in the Bible. Therefore, Augustine classifies three kinds of created entities: "those which exist and do not live; those which live and do not understand; and those which exist, live, and understand."[25] The first kind of created entity is the purely corporeal; the second is the living corporeal (including plants and animals); the third is human beings, who are corporeal, living and possess reason. So, all creatures receive numbers according to their size, magnitude, lifespan, and weight; the living creatures are endowed with the numbers of movement; only the rational creatures that exist, live, and understand receive the numbers of knowing/reasoning, in addition to the numbers of size, magnitude, weight, time, and movement.

Both the purely corporeal and the living corporeal are mortal because their magnitude and lifespan are in the temporal realm. However, the numerical proportion is immortal because it transcends the temporal realm and is exclusive and inviolable. Augustine argues that the human soul is immortal because it can recognize the immortal numerical principle. He thus attempts to sort out the intrinsic relationship between the soul, reason, the mind, and numbers. He ponders whether reason (*ratio*) can be called mind or spirit, or it is in the mind.[26] "If reason is one thing and mind another," Augustine posits, "surely we agree that only the mind can use reason, so that we have proved that he who has reason cannot be without a mind."[27] When the power of the rational soul recognizes the immortal science of numbers, "it has ventured to prove the soul immortal."[28] "Reason either is the soul itself or belongs to it, and that there is in reason nothing more excellent or dominant than numbers, or that reason is nothing other than number soliloquizes."[29] Therefore, when Augustine explores the spiritual world, the numerical ratios of the intelligible world serve as inviolable principles in his argument. In *The Magnitude of the Soul*, Augustine, by discussing at length the equal numbers of geometry, argues for unfailing equality as unfailing justice.

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[23] Ibid., 2.15.42.

[24] Wisdom 11: 21.

[25] Augustine, *De Libero Arbitrio*, 2.17.46.

[26] Augustine, *De Libero Arbitrio*, 1.9.19; cf. *De Immortalitate Animae*, 1.2, Patrologia Latina 32, J. P. Migne (ed.), Paris, 1841.

[27] Augustine, *De Immortalitate Animae*, 1.2.

[28] Augustine, *De Ordine*, 2.15.43.

[29] Ibid., 2.18.47.

In *The Immortality of the Soul*, by describing the immutable science of mathematics, Augustine argues for the immortality of human reason (*ratio*) and the mind: reason as an act of the mind can understand the immutable science of numbers. In so far as reason is an immutable power it can understand immutable science.[30]

Since the soul has been endowed with the numerical power of reasoning, it has the ability to discover the unchangeable power of numbers, but not to change it. When distinguishing music from grammar, Augustine illustrates that grammar can be altered and modified by authorities and culture; however, the principle of harmonic ratios cannot be changed by human power.[31] Virgil can change the pronunciation of a word, but he has no power to change the mathematical result of three times three according to his preference. Augustine repeatedly emphasizes that the numerical power in the divine order is not invented by the human mind, but discovered by the mind through reason, "Coming now to the science of number," he writes in Book II of his *De Doctrina Christiana*:

> it is clear to the dullest apprehension that this was not created by man, but was discovered by investigation. For, though Virgil could at his own pleasure make the first syllable of *Italia* long, while the ancients pronounced it short, however, it is not in any man's power to determine at his pleasure that three times three is not nine.[32]

Augustine applies the Pythagorean mathematic concept to developing his theological arguments because he believes that truth is common to everyone and no one can claim a private ownership of truth, stating, "the science of numbers was not ordained by men, but rather investigated and learned by them."[33] Augustine regards the true knowledge among pagans which can be harmonized with Christianity as treasures made by God. So, learning unchangeable truth from pagan philosophers, for him, is similar to the Israelites' taking silver and gold out of Egypt according to God's command:

> Furthermore, if those who are called philosophers, especially the Platonists, have said things by chance that are truthful and conformable to our faith, we must not only have no fear of them, but should instead appropriate them for our own use….The Egyptians not only had idols and crushing burdens which the people of Israel detested and from which they fled, but they also had vessels and ornaments of gold and silver, and clothing, which the Israelites leaving Egypt

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[30] Augustine, *De Immortalitate Animae* 5.9.

[31] Augustine, *De Musica*, 2.1.1.

[32] Augustine, *De Doctrina Christiana*, 2.38.56.

[33] Ibid.

secretly claimed for themselves as if for a better use. Not on their own authority
did they make this appropriation, but by the command of God.[34]

Definitely, gold and silver as natural resources, are not made by Egyptians, but only discovered and used by them. God commanded the Israelites to take these resources from the Egyptians and to make right use of them in their worship of the true God. Augustine refers to the biblical account of the ancient Israelites appropriating Egyptian wealth to justify the appropriation of the unchangeable knowledge that has been discovered by pagan philosophers. Augustine repeatedly states his conviction that "Whether numbers are considered in themselves, or as applied to the laws of figures, or of music, or of other motions, they have fixed laws which were not made by man, but discovered by the sagacity of intelligent men."[35]

In spite of different opinions about the essence of the world proposed by philosophers, Augustine realizes there is a common reason that drives philosophers to study the threefold division, natural, rational, and moral: it is because "none of them doubts that nature has some cause/principle (*principio*), science some method (logic), life some end and aim (ethics)."[36] Therefore, the assumption that the inviolable truth must exist in the three fields is the basis of undertaking philosophy, namely, seeking the unchangeable wisdom. Without this assumption, seeking truth would be an absurd will. The nature of truth must not only unchangeably transcend time and space, but can also be presented in time and space. The nature of numbers meets the criterion of the truth since it is neither corporeal nor temporal, but it is immutable and common to all generations. Augustine further argues: "Seven and three are ten, not only now, but forever. There has never been a time when seven and three were not ten, nor will there ever be a time when they are not ten. Therefore, I have said that the truth of number is incorruptible and common to all people."[37]

MUSIC: NUMERICAL SCIENCE

Augustine defines that *musica est scientia bene modulandi est movendi* (music is the science of good measuration and movement).[38] Regarding the source of Augustine's music

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[34] Ibid., 2.40.60.

[35] Ibid., 2.38.56.

[36] Augustine, *De Civitate Dei*, 11.24, Patrologia Latina 41, J. P. Migne (ed.), Paris, 1845.

[37] Augustine, *De Libero Arbitrio*, 2.8.21.

[38] Augustine, *De Musica*, 1.2.2-3.

definition, some scholars attribute this to Marcus Terentius Varro (116-27 BC).[39] Regrettably however, Varro's writing on music has been lost. In Censorinus' *De Die Natali*, where we find an early version of this definition: "*Igitur musica est scientia bene modulandi: haec autem est in voce: sed vox alias gravior mittitur, alias acutior.*"[40] Here, Censorinus' attention focuses on voice, and *bene modulandi* only indicates good measuring sound of higher or lower pitch. However, Augustine explains that his definition of music is not only about measuring the acoustical sound, but also about measuring both metaphysical forms by mathematical ratios and the end of music motions by ethical principles. He insists that the keywords: *scientia*, *modulandi*, *movendi* and *bene* must be applied for a complete music definition.

Scientia (Science)

"Science" in a modern sense is generally understood to relate to the study of nature, which is different from the disciplines of fine arts and humanities. This understanding coheres with Augustine's music definition. Music as a sub-discipline of mathematics is not a metaphor or outdated conception; undoubtedly, the ratios of the perfect intervals (unison 1/1, octave 1/2, fifth 2/3, and fourth 3/4) constitute the harmonic system which cannot be shaped by any culture. However, to regard music as a fine art is to assume that music is a product of social activities, that "[i]ts existence, function, and form are determined by the nature and values of the culture that enfold it."[41] Obviously, music modality, structure and styles are diverse in different cultures and times, and they are constantly shaped by cultures. However, what Augustine is concerned about is not the forms that can be shaped, but the unchangeable numeric ratios of harmonic system, which is the basis of music and can neither be created by man nor altered by culture or personal preference.

Myles W. Jackson, as a scientist, clarifies the relationship between music and science in ancient time, "Historically, music and science have been inextricably linked. Music provided (...) vast amounts of material for the investigations of natural philosophers...."[42] Underlying Augustine's music theory as a natural science, are three major Pythagorean resources: Plato's *Timaeus*, Claudius Ptolemy's *Harmonicorum Libri*

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[39] William R. Bowen, "St. Augustine in Medieval and Renaissance Musical Science", in Richard R. La Croix (ed.), *Augustine on Music—An Interdisciplinary Collection of Essays*, New York, Edwin Mellen Press, 1988, p. 31.

[40] Censorinus, *De Die Natali*, 10, 3, Apud J. L, S. Lechner, 1810, "Music is a good measuration of voice, in which some are lower, and some are higher."

[41] Ian Cross, "Music and Change: On the Establishment of Rules", in Peter Howell (ed.), *Musical Structure and Cognition*, London, Academic Press, 1985, p. 1.

[42] Myles W. Jackson, "Music and Science During the Scientific Revolution", *Perspective on Science*, vol. 9, no. 1, 2001, p. 115.

Tres, and Aristides Quintilianus' *On Music*. Based on the Pythagorean musical cosmology, Aristides combines astronomy, mathematics and the concept of the soul in his *On Music*. He divides the knowledge of music into two branches: the practical (including poetry and music performance) and the theoretical (including techniques and natural science, namely, arithmetic). Aristides defines the theoretical knowledge of music as a natural science: "Music is a science, certainly, in which exists sure and infallible knowledge."[43]

The word "science" derives from the Latin noun *scientia*, which covers a broad range of meanings, including "the knowledge of metaphysics" in an ancient philosophical context. The Pythagoreans regard learning the knowledge of metaphysics as a way to love wisdom. For them, wisdom does not concern knowledge of daily material things, but a kind of knowledge about the unchangeable essence and ultimate truth. The Pythagoreans regards the unchangeable numerical truth (arithmetic, music, geometry, and astronomy) as the only way leading to wisdom. This concept not only had a strong impact on the patristic age, but also exerted continuing influence throughout the Middle Ages. Boethius, one of the most influential music theoretician and Christian theologian in the Middle Ages, opens his *De Institutione Arithmetica* by introducing the quadrivium: "This four-way path will not hamper the skill of right reasoning. For wisdom, the comprehension of reality is of things which partake in immutable substance."[44] Further, he reasons that the wisdom of knowing the immutable substance cannot be acquired without the knowledge of these four disciplines, so he strictly excludes people who ignore these four disciplines from the realm of philosophy, says that "he who rejects these disciplines, these narrow ways to wisdom, I declare to him that he has no right to philosophize. For philosophy is the love of wisdom, he would have despised philosophy when he rejected these disciplines.[45]

As a Christian philosopher, Augustine is among the intellectual ancestors of Boethius. He accepted the Pythagorean disciplines without compromising his Christian faith. As has been mentioned, the unchangeable numerical truth, according to Augustine, is most compatible with Christian philosophy. In order to clarify the meaning of wisdom, Augustine draws a distinction between the connotations of wisdom and knowledge:

> If therefore this is the right distinction between wisdom and knowledge, the

[43] Aristides Quintilianus, *On Music*, trans. Thomas J. Mathiesen, New Haven, Yale University Press, 1983, p. 75.
[44] Boethius, *De Institutione Arithmetica*, 1.1, Gottfried Friedlein (ed.), Leipzig, B.G. Teubner, 1867.
[45] Ibid.

intellectual cognizance of eternal things belongs to wisdom, but the rational cognizance of temporal things to knowledge. It is not difficult to judge which is to be preferred or subordinated to which.[46]

From this differentiation, it can clearly be seen that, according to Augustine, music as one discipline of the four-way path to wisdom refers to the "intellectual cognizance" of eternity. Thus, when *scientia* is applied to the discipline of music, the connotation of this word is different from "the cognizance of temporal things."

Since harmonic ratio is an unchangeable science, Augustine holds that only human beings, the rational animals, are able to grasp the unchangeable truth. He compares human memories and senses with those of beasts and concludes that memories and senses of some beasts are even acuter than those of humans in some aspect, but animals cannot grasp unchangeable truth. Therefore, science cannot be obtained merely by human memory and senses; it is only attainable for the mind and exists in intelligence alone.[47] For Augustine, science sufficiently exists in the mind and it is independent from bodily skills, but no one can develop skills without a basis of science. Therefore, science which is in the mind should direct the bodily skill, and as a director of skill, it cannot be degenerated to merely a skill.[48] Furthermore, Augustine distinguishes human skill (*ars*) from mere imitation. He argues that human skill must consist in both imitation and reason. In comparison with the irrational animals, which are also able to imitate and memorize, human beings alone, according to Augustine, are capable of developing their skills/crafts. This is by the power of their reason. Augustine insists that if a performer only knows how to physically imitate others and his goal is solely the pursuit of money and fame, his imitating without reasoning is not different from the irrational behaviour of animals. For example, a nightingale is naturally able to sing; parrot can imitate man's singing without the use of reason.[49]

Modulandi and Movendi (Measuration and Movement)

Clarifying the meaning of science helps us to understand other keywords: *modulandi* and *movendi*, which relates to numbers and time. Augustine details the relationship between them: "Modulation is not improperly called a science of movement, for things must be arranged to move well. We cannot say something moves well if it has no *modus*."[50]

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[46] Augustine, *De Trinitate*, 12.15.25, Patrologia Latina 42, J. P. Migne (ed.), Paris, 1865.

[47] Augustine, *De Musica*, 1.4.7-8.

[48] Ibid., 1.4.5-9.

[49] Ibid., 1.6.11.

[50] Ibid., 1.2.3.

Therefore, the science of *bene modulandi* is about the science of *bene movendi*.[51] Augustine further explains what *modulandi* means, namely that, on account of the cosmic music perspective, all creatures which move in time must have been measured, and their measurement must be observed by God. He says, "For we cannot say anything moves well, if it does not contain *modum*."[52] On the one hand, *movendi* is essential to *modulandi*, for if nothing moves, nothing would be in measure; on the other hand, without *modulandi*, *movendi* cannot be, and nothing can be done.[53] Music as measured numbers moves in time, while the celestial motions measure the rhythm of time in a macroscopic fashion. These two kinds of motions serve as the best illustration for the relationship between movement and measurement, because none of them can move in a random manner. The inextricable relationship between measureation and celestial motions has been explained in Plato's creation myth in *Timaeus*. Plato states that the Creator measured three components: Same, Difference, and Being, and mixed them together, then divided them into seven portions mathematically.

> This is how he began the division: First he took one portion away from the whole, and then he took another, twice as large, followed by a third, one and a half times as large as the second and three times as large as the first. The fourth portion he took was twice as large as the second, the fifth three times as large as the third, the sixth eight times that of the first, and the seventh twenty-seven times that of the first.[54]

These seven numerical proportions can be illustrated by numbers as follows: 1-2-3-4-9-8-27. Except two, which is double one, every even number is double its previous even number; except one, every odd number is a triple of its previous odd number. The ratios of the first four numbers (1-2-3-4) constitute the four perfect harmonic intervals (1/1, 1/2, 2/3, 3/4). Plato uses these musical-harmonic ratios to conceive his creation myth in order to explain that these heavenly bodies were measured and set into harmonious motion by the Creator based on the musical-harmonic ratios. The length of these seven portions is bent into seven unequal circles by the Creator of the universe, who then sets the seven circles in motion: "He set the circles to go in contrary directions: three to go at the same speed, and the other four to go at speeds different from both each other's and that of the other three. Their movements, however, were all proportionate to each other."[55]

Ptolemy also applies the Pythagorean tuning system to explain the reason of

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[51] Ibid. "*Ergo scientiam modulandi iam probabile est esse scientiam bene movendi.*"

[52] Ibid., 1.2.3.

[53] Ibid.

[54] Plato, *Timaeus*, 35a-b, trans. Donald J. Zeyl, Indianapolis, Hackett Publishing Company, 2000.

[55] Ibid., 36d.

celestial motion. In Book III of *Harmonicorum*, he raises a question: "*Quomodo, per numerous, sumantur, suorum cujusque motuum rationes* (In what way, is reason active in the motions of itself and other things through numbers)?"[56] Ptolemy's explanation can be very helpful for understanding the interplay of mathematics, music, and astronomy:

> Under these conditions, the number of the square is 90, it is the mean between 120 (the number of the trine) and 60 (the number of the sextile); and it produces two ratios: 3/2 and 4/3, in accordance with the highest rank of consonant intervals, the fifth and fourth. As in music these first ranks of consonant intervals, the fifth and fourth, together make the octave identity. Similarly, as the two ratios have been mentioned, 3/2 and 4/3, produce the ratio 2/1 in conformity with the octave identity; therefore, in the same way, in comparison with the numbers of a whole circle, to divide 360 equally by the ratio of 4/1, it will come to 90. Definitely, the whole circle consists of the double octave of the perfect musical system.[57]

From the above passages of both Plato and Ptolemy, we can see that measurement is the prerequisite for motion and the harmonious motion of the planets is based on harmonic ratios. Augustine's conception on movement and measurement is based the Pythagorean tradition.

Bene (good)

Another keyword of the definition of music is *bene* (good). In order to explain the meaning of *bene*, Augustine first of all distinguish between *modulandi* and *bene modulandi*. He indicates that once things are measured, they can move well. The good movement itself only depends on measurement: "Because whatever keeps moving harmoniously in measured times and intervals can already be said to be moving well. If it is already pleasing, it is reasonable to call it modulation."[58] The modifier *bene* seems redundant. In the dialogue of *De Musica*, the Master thus asks whether it should simply be deleted.[59] He also suggests that the Disciple should check whether music, as science of movement, is a free motion of itself or refers to something else. "The motion, which is driven for its own sake and charm through itself, is free; while there is a servile way of motion, in which all things are not devoted to themselves but refer to something else."[60] If music moves for its own sake, the word *bene* would be redundant. If music motion

[56] Claudius Ptolemaeus, *Caudii Ptolemaei Harmonicorum Libri Tres*, 3.15, translated from Greek to Latin by Johannes Wallis, 1682 [Kessinger Publishing, LLC, 2010], Latin ed., p. 270.
[57] Ibid., pp. 270-71.
[58] Augustine, *De Musica*, 1.3.4.
[59] Ibid., 1.2.3.
[60] Ibid.

serves a purpose, *bene* would be necessary since the understanding that something possesses purpose is an ethical one. However, God, the Creator, is the only One whose motion is in the dominative, free positon and subject to nothing. The whole universe, as His creation, has been measured by God; its motion cannot be carried out without a purpose designed by the Creator. Therefore, regarding properly using harmony for a purpose, *bene* is necessary for the definition of music, as the Master explains:

> Yet it is possible for this harmony and measurement to please when they shouldn't. If one sings sweetly and dances gracefully and desires to frolic when the occasion requires solemnity; undoubtedly, harmonious mensuration is not used well. The motion which is good because it produces harmony can turn evil because it is improperly used. Therefore, it is one thing to measure, and another to measure well.[61]

If men intend to make art for the sake of art, on the one hand, they can divorce art from meaning and ethical principle; on the other hand, however, nothing can be carried out without a purpose, which is either good or empty (evil). Since *bene* is not a redundant adjective, accordingly, musical harmony does not merely refer to acoustical harmony, but also to the soul, that is, music must consist of the harmony of the soul and harmonious sound. It can be assumed that Augustine amplifies the meaning of *bene* in terms of ethics in light of *Timaeus*:

> And harmony, whose movements are akin to the orbits within our souls, is a gift of the Muses, if our dealings with them are guided by understanding, not for irrational pleasure, for which people nowadays seem to make use of it, but to serve as an ally in the fight to bring order to any orbit in our souls that has become disharmonized and make it concordant with itself.[62]

Aristides, whose philosophical treatise *On Music* probably serves as an example for Augustine, also confirms the Platonic view: "That not only the body of the universe but also the soul was organized and is considered through consonant numbers, the ancient and wise men affirmed confidently."[63] This musical cosmology, that the harmonious numbers of the universe correspond to that of the human soul, renders music the only mathematic truth relating to morality. *Bene* must therefore be considered the keyword in Augustine's definition of music.

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[61] Ibid.

[62] Plato, *Timaeus*, 47d.

[63] Aristides Quintilianus, *On Music*, trans. Thomas J. Mathiesen, New Haven, Yale University Press, 1983, p. 195.

THE HARMONIC *MODUS*: RATIOS OF THE FIRST FOUR NUMBERS

> When you see a great many kinds of sound in which distinct measures can be observed, and we admit this kind cannot be attributed to the art of grammar, don't you think that it is another discipline which contains harmonious numbers and skilful measurement? .... If I am not mistaken, this is what is called music.[64]

In *De Musica* II-V, Augustine mainly examines the unchangeable law which determines the ratios of longer and shorter temporal intervals in musical movement: "Since we are discussing numerically ordered movements (musical motion), I think we should first consider numbers themselves, and decide that whatever sure and fixed laws in the movements manifest to us shall be searched and apprehended."[65] The unchangeable laws of measurement that Augustine attempts to demonstrate by his investigation of the nature of numbers and musical motion are the ratios of the first four numbers, 1/1, 1/2, 2/3, 3/4. As mentioned previously, in his *Timaeus*, Plato demonstrates the Pythagorean cosmology that the progression of the first four numbers is the highest *Modus*, the measurement of harmonious motions of the whole universe. Augustine reiterates this cosmology and attempts to prove it in his *De Musica*.

Augustine first observes the sequential relationship between the first three numbers, one, two, and three. The number three is introduced as the first perfect odd number, which contains the perfect progression, a beginning (one), middle (two), and end (three). This progression cannot be found in the number one: "One has no middle and end, because there is only a beginning, or rather it is a beginning because it lacks a middle and end."[66] The middle and end cannot be found within the number two because it combines two beginnings. Two beginnings of numbers added together make the whole and perfect number three. Augustine describes the perfect relationship between one, two, and three: Firstly, these three numbers represent a perfect whole—the beginning, the middle, and the end. Secondly, one and two result in three (1+2=3). Thirdly, one and two are followed by three in progression (1-2-3). This perfect harmony in the first three numbers cannot be found in any other three consecutive numbers.[67]

Augustine further proves that the number 4 is the first whole even number, and it is also the determinant number of the fourfold progression *modus* (1-2-3-4). The number 4 follows the progression of 1-2-3, and it also consists of 1 and 3, or twice the amount of 2. In the progression of 1-2-3, 1 and 3 are extremes, while 2 is the mean. The proportional

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[64] Augustine, *De Musica*, 1.1.1.

[65] Ibid., I.11.19.

[66] Augustine, *De Musica*, 1.12.21.

[67] Ibid., 1.12.20-22.

agreement of the extremes with the mean is called analogia (ἀναλογια) in Greek or proportion (*proportionem*) in Latin.[68] Augustine emphasizes that "not only one, two, three, but also one, two, three, four is the most closely connected progression of numbers."[69] In the progression of 1-2-3-4, another perfect relationship can be discovered: two extremes, 1 and 4, result in 5; two means, 2 and 3 is also 5; and the number 5 follows 4 consecutively. This perfect harmony cannot be found in the progression of any other four consecutive numbers. Therefore, Augustine holds that the first four numbers (1-2-3-4) contain the nature of all numbers; their progression is the determinative *modus* for the order of all other numbers:

> Considering the rest of numbers, you should not think that they have no characteristics of the quaternary numbers, from which all other numbers are derived; the connection of the quaternary numbers from one to four is a determinative numerical collection and the most beautiful progression of *modus*.[70]

Augustine highlights the progression of one to four in the conclusion of *De Musica* by raising a question: "From where, then, is the measure of this progression of one to four?... Where, I ask, do these things come from, if not from the highest and eternal rule of numbers, likeness, equality, and order?"[71]

The observation of the special progression of one to four is neither a meaningless number game, nor an innovation discovered by Augustine. The progression from one to four is the foundation of the Pythagorean tuning system. The ratios of these first four numbers constitute four perfect harmonic intervals: one to one (1:1) is the unison; one to two (1:2) is the octave; two to three (2:3) is the fifth; and the ratio of three to four (3:4) is the fourth. The Pythagorean tuning system is calculated according to the ratios of 1:2 and 2:3. Harmonic intervals are produced consecutively by multiplying the ratios of the fifth and the octave. When the note interval starts from C, the intervals can be illustrated as follows:

G^b: diminished fifth $\qquad \left(\frac{2}{3}\right)^6 \times \left(\frac{2}{1}\right)^4 = \frac{1024}{729}$

D^b: minor second $\qquad \left(\frac{2}{3}\right)^5 \times \left(\frac{2}{1}\right)^3 = \frac{256}{243}$

A^b: minor sixth $\qquad \left(\frac{2}{3}\right)^4 \times \left(\frac{2}{1}\right)^3 = \frac{128}{81}$

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[68] Ibid., 1.12.23.
[69] Ibid.
[70] Augustine, *De Musica*, 1.12.24.
[71] Ibid., 6.17.57.

E^b: minor third $\qquad \left(\frac{2}{3}\right)^3 \times \left(\frac{2}{1}\right)^2 = \frac{32}{27}$

B^b: minor seventh $\qquad \left(\frac{2}{3}\right)^2 \times \left(\frac{2}{1}\right)^2 = \frac{16}{9}$

F: perfect fourth $\qquad \frac{4}{3}$

C: Octave $\qquad \frac{2}{1}$

C: unison $\qquad \frac{1}{1}$

G: perfect fifth $\qquad \frac{3}{2}$

D: major second $\qquad \left(\frac{3}{2}\right)^2 \times \frac{1}{2} = \frac{9}{8}$

A: major sixth $\qquad \left(\frac{3}{2}\right)^3 \times \frac{1}{2} = \frac{27}{16}$

E: major third $\qquad \left(\frac{3}{2}\right)^4 \times \left(\frac{1}{2}\right)^2 = \frac{81}{64}$

B: major seventh $\qquad \left(\frac{3}{2}\right)^5 \times \left(\frac{1}{2}\right)^2 = \frac{243}{128}$

$F^{\#}$: augmented fourth $\quad \left(\frac{3}{2}\right)^6 \times \left(\frac{1}{2}\right)^3 = \frac{729}{512}$

These notes starting from C are sequenced in the order of the perfect fifth. The ratio between every two adjacent notes is the perfect fifth ($G^b - D^b - A^b - E^b - B^b - F - C - G - D - A - E - B - F^{\#}$). The perfect fifth and the perfect fourth make an octave both technically and numerically ($3/2 \times 4/3 = 2/1$). The table clearly shows that all harmonic intervals of the Pythagorean tuning have been determined by the perfect fifth and measured by the ratio of octave. In *De Libero Arbitrio*, Augustine emphasizes the ratio of one to two (1:2), stating, "Through all of the rest of the numbers you will find the same thing that is found in the first pair of numbers, one and two."[72] In his *De Trinitate*, Augustine regards the ratio of the octave (1:2) as the *modus* of everything, and, by the power of this ratio, God created the world, measured the world, and redeemed the world. Human beings have the ability of singing in and appreciating harmony because the power of octave has been built into the human mind by God.

> The ratio of Christ's bodily death and resurrection corresponding to the death and resurrection of both the human soul and body. The ratio of one to two can be called agreement, or accord, or singing or, more suitably, consonance, which is

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[72] Augustine, *De Libero Arbitrio*, 2.8.23, "*Atque ita per omnes caeteros reperies hoc quod in prima copula numerorum, id est uno et duobus inventum est, ut quotus quisque numerus est ab ipso principio, totus post illum sit duplus eius.*"

the great power in all composite structures, or better perhaps, the "coadaptation of creatures." I am reminded that the word co-adaptation is harmony (ἁρμονία) in Greek. Though the quantity of harmony cannot be shown in space, the power of one to two is found especially in us since it is naturally implanted in us (and by whom, unless by Him who created us?) Therefore, even the ignorant cannot fail to perceive it, whether when singing themselves or hearing others.[73]

As has been discussed, the significant meaning of the sequence of the first four numbers (1-2-3-4) has been illustrated by Plato's creation myth in *Timaeus*.[74] Plato describes that how the Creator measured the seven portions of the mixture of the same, the different, and the being in a progression of seven numbers, 1-2-3-4-9-8-27. In this progression, there are three even numbers and three odd numbers, not considering the number one. Except for two, which is the double of one, and three, which is the triple of one, the even numbers are double their previous even numbers, while the odd numbers are triple their previous odd numbers. The relationship that triple and double relate to the same number cannot be found among any other three numbers except for the ratios between one, two, and three (1-2-3). Furthermore, the relationship of double and triple is perfectly manifested by the consecutive progression of the first four numbers (1-2-3-4), and the pattern of this consecutive relationship cannot be found in the progression of any other four numbers in the order of doubling the evens and tripling the odds, (e. g. 9-8-27-16, these four numbers are not in a consecutive progression). The ratios of these seven numbers, 1-2-3-4-9-8-27, constitute precisely the Pythagorean tuning system, which was also explained by Plato in his *Timaeus*:

> These connections produced intervals of 3/2, 4/3, and 9/8 within the previous intervals. He then proceeded to fill all the 4/3 intervals with the 9/8 interval, leaving a small portion over every time. The terms of this interval of the portion left over made a numerical ratio of 256/243.[75]

The Pythagorean system can be rearranged according to the five ratios 1/1, 1/2, 2/3, 3/4, 8/9 in *Timaeus*, yet the results of ratios are exactly the same:

G^b: diminished fifth $\qquad \left(\dfrac{8}{9}\right)^3 \times \dfrac{2}{1} = \dfrac{1024}{729}$

D^b: minor second $\qquad \left(\dfrac{8}{9}\right)^2 \times \dfrac{4}{3} = \dfrac{256}{243}$

A^b: minor sixth $\qquad \left(\dfrac{8}{9}\right)^2 \times \dfrac{2}{1} = \dfrac{128}{81}$

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[73] Augustine, *De Trinitate*, 4.2.4.
[74] Plato, *Timaeus*, 36b-36d.
[75] Ibid.

E$^{\flat}$: minor third $\qquad\left(\dfrac{8}{9}\right)^{1}\times\dfrac{4}{3}=\dfrac{32}{27}$

B$^{\flat}$: minor seventh $\qquad\left(\dfrac{8}{9}\right)^{1}\times\dfrac{2}{1}=\dfrac{16}{9}$

F: perfect fourth $\qquad\dfrac{4}{3}$

C: Octave $\qquad\dfrac{2}{1}$

C: unison $\qquad\dfrac{1}{1}$

G: perfect fifth $\qquad\dfrac{3}{2}$

D: major second $\qquad\left(\dfrac{9}{8}\right)^{1}\times\dfrac{1}{1}=\dfrac{9}{8}$

A: major sixth $\qquad\left(\dfrac{9}{8}\right)^{1}\times\dfrac{3}{2}=\dfrac{27}{16}$

E: major third $\qquad\left(\dfrac{9}{8}\right)^{2}\times\dfrac{1}{1}=\dfrac{81}{64}$

B: major seventh $\qquad\left(\dfrac{9}{8}\right)^{2}\times\dfrac{3}{2}=\dfrac{243}{128}$

F$^{\#}$: augmented fourth $\qquad\left(\dfrac{9}{8}\right)^{3}\times\dfrac{1}{1}=\dfrac{729}{512}$

According to harmonic order, ratios can be arranged in the following sequence: unison 1/1, octave 2/1, perfect fifth 3/2, perfect fourth 4/3, major second 9/8, minor seventh 16/9, major sixth 27/16, minor third 32/27, major third 81/64, minor sixth 128/81, major seventh 243/128, minor second 256/243, augmented fourth 729/512, diminished fifth 1024/729. The musical interval ratios are the results of the multiplied ratios of the first six numbers when 9/8 is multiplied and measured by the ratios of the first four numbers 1/1, 1/2, 2/3, 3/4 respectively. These numerical interval ratios are clearly not just random results, but naturally exist in the inviolable order of doubling the evens and tripling the odds:

1-2-3-4-9-8-27-16-81-32-243-64-729-128-729×3-256-729×3^{2}-512-729×3^{3}-1024

When the ratios of major and minor, augmented and diminished connect together, the combination of the numbers of their names make nine while their ratios result in the octave 2/1:

Unison and Octave $\qquad\dfrac{1}{1}\times\dfrac{2}{1}=\dfrac{2}{1}$

Perfect fifth and perfect fourth $\qquad\dfrac{3}{2}\times\dfrac{4}{3}=\dfrac{2}{1}$

Major second and minor seventh	$\dfrac{9}{8}\times\dfrac{16}{9}=\dfrac{2}{1}$
Major sixth and minor third	$\dfrac{27}{16}\times\dfrac{32}{27}=\dfrac{2}{1}$
Major third and minor sixth	$\dfrac{81}{64}\times\dfrac{128}{81}=\dfrac{2}{1}$
Major seventh and minor second	$\dfrac{243}{128}\times\dfrac{256}{243}=\dfrac{2}{1}$
Augmented fourth and diminished fifth	$\dfrac{729}{512}\times\dfrac{1024}{729}=\dfrac{2}{1}$

Quite evidently, the most powerful ratio is one to two for it determines all harmonic ratios. This harmonic system also is called the octave system. The octave is a scientific phenomenon. It is an interval between the beginning note and the final note, which is also the beginning of the next scale in a seven-tone mode. The ratio between two notes of octave is single to double (1:2), therefore, the frequency of the pitch of the latter is double that of the former. Augustine understands well the harmonic principle that the harmonic pitches are determined by the ratio of octave and any notes apart from this harmonic standard would be disharmony. This scientific principle, for Augustine, offers sound evidence to regard the power of one to two as the determined power for all numbers and all harmonic power of the world both scientifically and figuratively. If God created the world with the power of numbers and measured every creature's form according to harmonic order, thus determining their nature, the fundamental power cannot be other than the ratio of one to two.

> The high and low voices are harmonious through this ratio (1:2). Therefore, whoever departs from it, makes disharmony which extremely offends not the music theory that most people lack, but the very sense of our hearing. To demonstrate this, no doubt needs a long discourse; but anyone who knows the ratio, may easily discover it by listening to the rightly ordered monochord.[76]

The rightly ordered monochord is an ancient scientific instrument, which illustrates the mathematical standard of harmony, and its Greek name is kanon (κανόνας), it means law and standard. "Music, mathematics, and astronomy were inexorably linked in the monochord."[77] Monochord, the only one string instrument, for Augustine, indicates Jesus, the *Monarchia* (the absolute Rule of God). According to the harmonic power of the Monochord, the human soul can be tuned to the right order.

When Augustine probes into the proportional rhythms of longer and shorter time

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[76] Augustine, *De Trinitate*, 4.2.4.
[77] Siemen Terpstra, "An Introduction to the Monochord", *Alexandria 2: The Journal of the Western Cosmological Traditions*, vol. 2, 1993, p. 137.

COSMOS AND HISTORY

in musical motion, he attempts to demonstrate that not only has the relationship of all harmonic intervals been built up by the ratios of the first four numbers, 1-2-3-4, but the harmony of rhythm is also determined by the ratios of the four numbers. Augustine differentiates the theory of music rhythm from Latin grammar by clarifying the ratios of syllables, metrical feet, rhythm, meter and verse. He raises a question intend to emphasize the exclusive powerful ratios in the sequence of one to four (1-2-3-4): "Do you see there could be no other progression than from one to four either in feet or times?"[78]

> *Master*: Then, since feet consist of syllables, that is, of distinct and articulate movements of sound, and syllables are extensions of times, don't you think the progression within the foot should go to four syllables, just as the progression of feet and times goes as we have seen to four? *Disciple*: I feel about it as you say and I know it is perfectly reasonable.[79]

He tirelessly defends his assertion that the progression of one to four (1-2-3-4) is the determinative power in the ratio of feet, rhythm, meter and verse.

> And so, in the rule of numbers going from one to four, there is nothing nearer each one than itself. And, therefore, those feet take precedence whose parts are in relation of equality to each other (1:1). Then the union of single and double emerges in one to two (1:2); the sesquialter union in two and three (2:3); and the sesquitertian in three and four (3:4).[80]

According to the ancient metrical system based on Latin language, metrical foot is the smallest unit of rhythm. The structure of one foot is no more than four syllables and no less than two. Therefore, there are three types of foot: the two syllable-foot, the three syllable-foot, and the four syllable-foot. Firstly, there are four kinds of ratios of the times regarding two syllables in the shortest foot: two shorts syllables (1:1), a short and a long (1:2), a long and a short (2:1) and two longs (2:2). Secondly, "because every foot must have two parts to be compared with each other by means of some ratio,"[81] in the three-syllables foot there are three kinds of ratios of time: a long and two shorts (2:1:1), two shorts and a long (1:1:2) and a short and a long and a short (1:2:1). Thirdly, the ratios of times in the four syllable-foot can be found as these: one long and three shorts (2:1:1:1), three shorts and one long (1:1:1:2), two longs and two shorts (2:2:1:1), two shorts and two longs (1:1:2:2), a short and a long and a short and a long (1:2:1:2), a long and a short and a long and a short (2:1:2:1), a short and two longs and a short (1:2:2:1), a short

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[78] Augustine, *De Musica*, 2.4.5.
[79] Ibid.
[80] Augustine, *De Musica*, 2.10.19.
[81] Ibid., 2.5.6.

and three longs (1:2:2:2). Augustine anatomizes the basic elements of rhythmic ratios in order to demonstrate that the ratios of the fourfold progression determine the fundamental principle of rhythm in western music.

> *Master*: Have you sufficiently considered to what extent that progression from one to four, the demonstration of numbers, is also true for feet? *Disciple*: I certainly prove that these ratios of four numbers exist in feet as in the others.[82]

Either the times of rest-intervals in a verse or the numbers in a combination circle cannot be less than one or more than four: "For this is that very same measured progression so much has already been said about. In any foot no arsis or thesis takes more than four times."[83] In *De Musica*, Augustine repeatedly emphasizes the quaternary ratio:

> Meter is bounded by a fourfold number of times for its beginning, and verse by a fourfold number of feet for its beginning. So this quaternary ratio is kept, and meter evidently shares with verse its manner of expansion in feet, verse with meter in times.[84]

Based on these mathematic principles of harmony, Augustine illustrates how the ratios of the first four numbers (1-2-3-4) not only determine the harmonic intervals, but also powerfully condition the rhythm of musical motion. Therefore, in music measurement, Augustine concludes that "you must consider the fourfold progression."[85]

CONCLUSION: BEAUTY IN NUMERICAL ORDER

> To investigate the beauty of corporeal forms, it is that numbers are contained in space; to investigate the beauty of movement in substance, it is that numbers move around in time.[86]

For Augustine, the unchangeable, harmonic principle is the highest modus/glory of God which bestows beauty, truth, goodness, and harmony upon the created world as the purpose of its existence. Discussion on the unchangeable harmonic principle of God inevitably relates to the issues of origin, morality, meaning and destiny. Thus, cosmology is inevitably involved in theological and philosophical discussions, bringing epistemology and teleology into play as well, in order to explain the origin of the cosmos and the purpose of its existence. In the realm of teleology, human intelligence

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[82] Ibid., 2.7.14.
[83] Ibid., 3.8.18.
[84] Ibid., 3.9.21.
[85] Ibid., 3.9.20.
[86] Augustine, *De Libero Arbitrio*, 2.16.42.

can taste the beauty of "the moral harmony and metaphysical splendour"[87] by reasoning about the purpose of the divine order because the transcendent ontological beauty of God not only surpasses the realm of the senses, but is also far beyond human cognition. Augustine believes that order originates in God; by order, God governs all things and establishes a relationship with human beings, as he says: "Order is that which will lead us to God, if we hold to it during life; and unless we do hold to it during life, we shall not come to God."[88] Nevertheless, according to the Pythagorean conviction that mathematical truth is the best way to know the unchangeable order,[89] Augustine thus also recognizes that unchangeable order is manifested by the quadrivium, as he says:

> Now in music, in geometry, in the movements of the stars, in the fixed ratios of numbers, order reigns in such manner that if one desires to see its source and its very shrine, so to speak, he either finds it in these, or he is unerringly led to it through them.[90]

Except for the numerical-musical order, nothing is the unique means that is operated by both mathematical ratios and ethical principles. Therefore, the Pythagorean philosophers in particular used mathematic-harmonic order as the fundamental theories to explore the origin and the ethical meaning of the cosmos. Andrew Barker, a musicologist recognizes that Plato's astronomy and harmonics are idiosyncratic, because "his astronomy is not a science that aims to describe accurately the observed movements of the heavenly bodies, and the task of his harmonics is not to analyse any musical systems in actual use."[91] As a Pythagorean philosopher in respect of his musical cosmological view, Plato's interest in natural science is not for the sake of scientific research in the modern sense, but for the purpose of knowing the cause (truth) and understanding the meaning (ethical good) of the world. In his *Republic*, Plato states that the purpose of music is nothing other than pursuing beauty and goodness: "Yet, it is useful in the search for the beautiful and the good! Pursued for any other purpose, it is useless."[92] Further, in his *Phaedo*, he claims that "[p]hilosophy is the

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[87] Umberto Eco, *Art and Beauty in the Middle Ages*, trans. Hugh Bredin, New Haven, Yale University Press, 1986, p. 1.

[88] Augustine, *De Ordine* (*Divine Providence and the Problem of Evil*), 2.9.21., trans. Robert P. Russell, in Ludwig Schopp (ed.), *The Fathers of the Church*, vol. 1, New York, Cima, 1948.

[89] Ibid., 2.20.53-54.

[90] Ibid., 2.5.14.

[91] Andrew Barker, *Greek Musical Writings II: Harmonic and Acoustic Theory*, Cambridge, Cambridge University Press, 1989, p. 53.

[92] Plato, *Republic*, 231c5, trans. C.D.C. Reeve, Cambridge, Hackett Publishing, 2004, p. 226.

greatest music, and that was what I was doing."[93] By investigating the unchangeable mathematic-harmonic ratios, Plato recognized the purpose of the Creator is harmony: "Logos (*ratio*) was designed for this very purpose—it plays the greatest part in its achievement. All such composition [music] as lends itself to making audible, musical sound is given in order to express harmony, and so serves this purpose as well."[94]

Augustine agrees with this philosophical approach to the highest Beauty. By investigating the unchangeable mathematical truth in music, Augustine intends to seek the ultimate goodness, beauty, and harmony. Based on the Pythagorean concept that the mathematical truth is the divine arts of God leading people to love wisdom (*philo sophia*), Augustine holds that number and wisdom are one and the same because the wisdom of God orders things in number and assigns numbers to all things.[95] In his *Retractationum*, although Augustine distinguishes between the philosophical intelligible world and the spiritual realm (the kingdom of God),[96] he corrects his early opinion stated in *Contra Academicos* that "love of the beautiful, the good" (*philo calia*) is a sister of "love of wisdom" (*philo sophia*), arguing instead that they are the same: "To love beauty in things incorporeal and sublime is the same thing as loving wisdom, and in no way are they, as it were, two sisters."[97] By now, we can see that according to Augustine, the supreme wisdom and beauty, which should be loved by humans, are the same. It is quite clear that music as a discipline, which relates to numbers, beauty, and wisdom, was understood as an essential part of *philo Sophia* in the Pythagorean tradition. Music as an incorporeal and measured numerical motion serves as the best means to illustrate metaphysical beauty, as Augustine argues in *De Musica*: *Haec igitur pulchra numero placent, in quo iam ostendimus aequalitatem appeti* (Things, in which we desire to express equality, please us because of beautiful numbers).[98] In other words, the incorporeal numbers are the basis of the beauty which can be perceived by the senses.

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[93] Plato, *Phaedo*, 61a, trans. Alex Long, Cambridge, Cambridge University Press, 2010, p. 46.

[94] Plato, *Timaeus*, 47d.

[95] Augustine, *De Libero Arbitrio*, 2.11.30.

[96] Augustine, *Retractionum*, 1.3.2, Patrologia Latina 32, J. P. Migne (ed.), Paris, 1841.

[97] Ibid., 1.1.3.

[98] Augustine, *De Musica*, 6.13.38.

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