THE FOUR ELEMENTS AND THEIR CHARACTERISTICS ACCORDING TO THE SCHEMA IN THE EARLY MEDIEVAL ANONYMOUS FRAGMENT *DE QUATTUOR ELEMENTIS*

Marek Otisk

ABSTRACT: The paper is an attempt to interpret a schema that is part of an early medieval fragment known as the *Excerptum de quattuor elementis*. In this schema, the individual elements (fire, air, water, earth) are presented as the elements which form the material world using several characterisations typical of the period: as the sum of their natural physical properties in accordance with the Platonic and Aristotelian traditions, as geometric shapes (to put it simply, using the so-called Platonic solids) and as numerical values connected by a 2:1 ratio. The aim of this paper is to propose a possible interpretation of the numerical values and connections found in this schema using contemporary texts and diagrams (especially Calcidius and Isidore of Seville, as well as other sources available at the time), which would include a consistent yet complex characterisation of the elements, their properties and their mutual interconnectedness, as found in the fragmentary text of the *Excerptum de quattuor elementis* and in the schema itself.

KEYWORDS: Elements; Materia; Early Middle Ages; Scientific diagram; Plato; Aristotle; Calcidius; Boethius; Isidore of Sevilla

I. INTRODUCTION

In a short anonymous text published by the editor R. A. B. Mynors under the name *Excerptum de quattuor elementis*[1] as an appendix to Cassiodorus' *Institutiones*, which most likely originated between the second half of the 6th and the end of

[1] *Excerptum de quattuor elementis*, ed. R. A. B. Mynors, in Cassiodorus, *Institutiones*, Oxford, Clarendon Press, 1937, pp. 167–168 (hereafter cited as *EDQE*).

the 8th century,[2] there is a diagram showing the nature of and ties between the four basic elements – *ignis* (fire), *aer* (air), *aqua* (water), and *terra* (earth) – which form the changeable material world. This schema includes several different views on the elements which continued to be an important topic of discussion (not only) in the Early Middle Ages, mainly as a result of the legacy of Platonic and Aristotelian perspectives.

The schema characterises the elements through:

1. Combinations of the four basic Aristotelian properties: *siccus/aridus* (dry, arid) or *humidus* (wet, moist, humid), and *calidus* (warm, hot) or *frigidus* (cold, frigid);
2. Combinations of the six basic Platonic properties: *mobilis* (mobile, moveable) or *immobilis* (immobile, immoveable), *subtilis* (subtle, sharp) or *corpulentus* (corpulent, blunt), and *acutus* (acute, thin) or *obtusus* (obtuse, thick);
3. Three-dimensional geometric shapes, mainly regular polyhedrons: *piramis* (pyramid or tetrahedron), *sphera* (sphere representing the *octohaedron* or octahedron), *icosahedron* (icosahedron), *cybos* (cube, hexahedron);
4. Numerical values (12, 24, 48, 96) representing a 2:1 ratio and their interconnection by means of mutual multiples (576, 1152, 2304).

The schema thus includes the natural and physical properties of the elements, along with their geometric and arithmetic representation, which clearly combine the Platonic and Neoplatonic teachings on the elements based primarily on the dialogue *Timaeus* with the Aristotelian teachings on the elements described especially in *On Generation and Corruption*.

This anonymous early medieval fragment has already received some attention[3] and this study aims to follow up on this by providing a multi-faceted contribution to the debate on the importance of graphic illustration: in particular,

[2] Cf., e.g., Irene Caiazzo, 'Filosofia della natura e fisica elementare nell'alto medioevo', in *La conoscenza scientifica nell'Alto Medioevo*, Spoleto, Fondazione CISAM, 2020, pp. 1069–70.

[3] See, e.g., Barbara Obrist, *La cosmologie médiévale. Textes et images. I. Les fondements antiques*, Firenze, Sismel – Edizioni del Galluzzo, 2004, pp. 284–9; John E. Murdoch, *Album of Science. Antiquity and the Middle Ages*, New York, Scribner's Sons, 1984, pp. 350–1; Léon Pressouyre, 'Le cosmos platonicien de la cathédrale d'Anagni', *Mélanges d'archéologie et d'histoire*, vol. 78, no. 2, 1966, pp. 570–2; Richard McKeon, 'Medicine and Philosophy in the Eleventh and Twelfth Centuries: The Problem of Elements', *The Thomist: A Speculative Quarterly Review*, vol. 24, no. 2–4, 1961, pp. 223–4; Caiazzo, 'Filosofia della natura…', pp. 1069–71; Annemieke R. Verboon, *Lines of Thought. Diagrammatic representation and the scientific texts of the arts faculty, 1200–1500* [doctoral thesis], Leiden, Leiden University, 2010, pp. 119–20.

the paper will focus on the reasons that may have led the unknown author to identify individual elements with certain numerical values (linking geometry and arithmetic) and, on the basis of this, formulate a hypothesis to explain the suggested relationship between multiples as a fusion of mathematical and physics-based interpretations of the properties of the elements. These primary aims of study will be preceded by a framing of the teachings on the four elements and their properties – as presented in the schema and the *EDQE* text itself – in their contemporary context, using the texts of Calcidius, Boethius and Isidore of Seville. Other contemporary sources, such as Martianus Capella, Macrobius, Bede the Venerable etc., will also be acknowledged.

II. *EXCERPTUM DE QUATTUOR ELEMENTIS* AND ITS SCHEMA

EDQE's brief treatise is, at its core, based exclusively on the Platonic doctrine of the elements. The text opens (*EDQE* 167,19–20) with a declaration of the intention to show how the so-called middle elements (*media mundi corpora*) – water and air – obtain their nature (*ingenium*) and character (*vigor*) from numbers (*numeri*). The author combines the numerical geometric sequence represented by three-dimensional geometric shapes from Plato's *Timaeus* with a natural representation of the basic properties of the elements.[4] This is why they mention that the two middle elements (water and air), which are three-dimensional formations, take the lengths of two of their three sides (*latera*) from the element closest to them (*vicinia*) and the length of the remaining third side from the element farthest from them (*extimum*), thus they contain the properties (*qualitates*) of both extreme elements – fire and earth (*EDQE* 167,21–4).

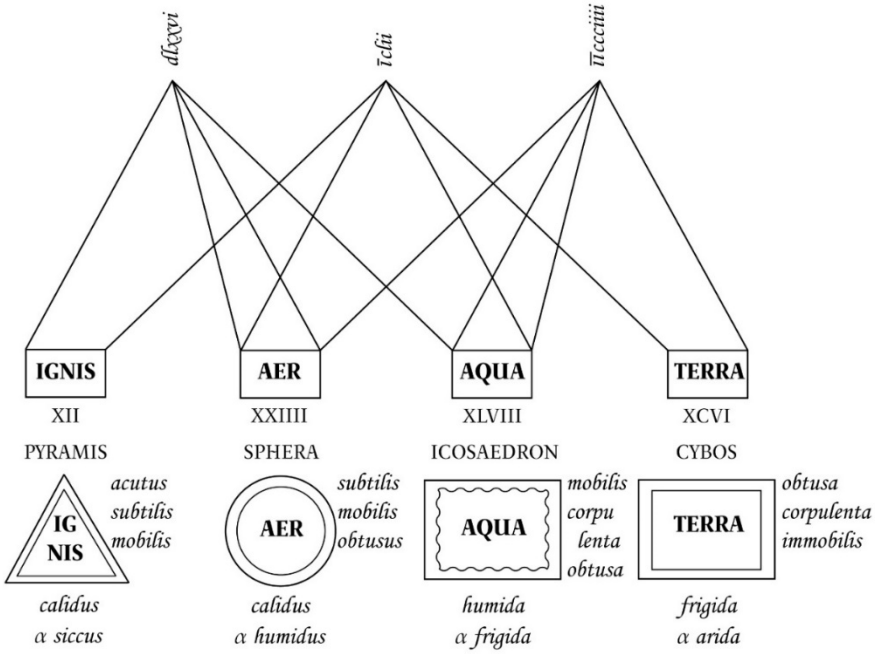
Earth and fire are characterised by their opposing properties (*controversa*) – earth is obtuse (*obtusa*), corpulent (*corpulenta*) and immobile (*immobilis*), while fire is acute (*acutus*), subtle (*subtilis*) and mobile (*mobilis*). Both middle elements (water

[4] On the knowledge and usage of this Plato's dialogue in the early Middle Ages, including the frequency of copying of Latin translations, see, e.g., Henry Mayr-Harting, *Church and Cosmos in Early Ottonian Germany. The View from Cologne*, Oxford, Oxford University Press, 2007, pp. 153–61; Ernst H. Kantorowicz, 'Plato in the Middle Ages', *The Philosophical Review*, vol. 51, no. 3, 1942, pp. 319–21; Anna Somfai, 'The Eleventh-Century Shift in the Reception of Plato's *Timaeus* and Calcidius's *Commentary*', *Journal of the Warburg and Courtauld Institutes*, vol. 65, no 1, 2002, pp. 3–21 and others. For a summary of translations of *Timaeus* into Latin and the Latin reception of this work in the antiquity and the nascent medieval Christian culture, see in particular Christina Hoenig, *Plato's* Timaeus *and the Latin Tradition*, Cambridge, Cambridge University Press, 2018, pp. 44–101 (Cicero), pp. 117–58 (Apuleius), pp. 168–213 (Calcidius) and pp. 227–78 (Augustine).

and air) are needed in order for all three properties to become their opposite (*EDQE* 167,24–168,2), as these middle elements always take on two properties (*virtutes*) from the closer element and the third from the more distant one. Water is the element that is closest (*proxima*) to earth, so it takes on its corpulence and obtuseness while borrowing mobility from fire. Air, in turn, is closer to fire and it shares with it its mobility and subtlety while receiving obtuseness from earth (*EDQE* 168,2–14). This interconnectedness of the properties of opposite elements through the middle elements is supposed to be clarified through lines (*virgulae*) indicated in the attached diagram, the obliqueness of which shows how the properties of the elements are connected and how further variations are produced (*EDQE* 168,15–20).

The schema is as follows (Fig. 1):[5]

FIG. 1 – SCHEMA FROM THE *EXCERPTUM DE QUATTUOR ELEMENTIS*



[5] For Fig. 1, in addition to the Mynors edition, I mainly used the following manuscripts: Karlsruhe, Badische Landesbibliothek, Aug. perg. 106, fol. 43v; Bern, Burgerbibliothek, Cod. 234, fol. 45r; Città del Vaticano, Biblioteca Apostolica Vaticana, Reg. lat. 123, fol. 129r; Bern, Burgerbibliothek, Cod. 212, fols. 90r–v.

III. THE PHYSICAL PROPERTIES OF THE ELEMENTS

Writing around the same time, Isidore of Seville (circa 560–636) provides a relatively clear discussion of the elements that make up the material world. In *Etymologies* XIII, 3, 1[6] he briefly mentions the initial formation of a formless substance (*ὕλη*, *materia*, *silva*) through the elements (*στοιχεῖα*, *elementa*) and then goes on to explain the Greek term for them in the following paragraph. The reason why Greeks call them *στοιχεῖα* is because they come together (*convenire*) based on harmonic concord (*concordia*) and mutual communality (*communio*). All elements intertwine through a natural order (*ratio naturalis*) – sometimes they derive their origin (*origo*) from fire and gradually return to the earth or, at other times, they start from earth and end at fire. This is because fire turns into air by becoming corpulent, water is formed when air becomes obtuse and earth is formed when water becomes immobile; in contrast, earth becomes water by becoming mobile, water becomes air by becoming acute, which then becomes fire as it becomes subtle (*Etym.* XIII, 3, 2).

The elements are therefore interconnected and collectively transition from one to the other. The Greek term for this interconnectedness is *συζυγία*, which Isidore also mentions in connection with grammar and metric feet that are longer than four syllables (*Etym.* I, 17, 1; or I, 17, 20) – which are known as syzygies (*syzygiae*). Martianus Capella (beginning of the 5th century CE) expresses similar ideas in the encyclopaedic compendium *De nuptiis Philologiae et Mercurii*, where he presents the Latin equivalent of *copula*, as that is the term for a connection (*conexio*) of two feet that appear to be dissimilar (*dissimiles*).[7] However, the term has also been used in parallel to denote the interrelationships between the elements and, in Latin texts, *syzygia* is sometimes referred to as *coniugatio* or *coniunctio*.[8]

Isidore deals with the issue of elements as the basic components of the material world in more detail in *De natura rerum*, where he writes that there are four such elements and he also lists both their Platonic and Aristotelian properties.[9]

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[6] Isidore of Seville, *Etymologiarum sive Originum libri XX*, ed. W. M. Lindsay, Oxford, Oxford University Press, 1911 (hereafter cited as *Etym.*).

[7] Martianus Capella, *De nuptiis Philologiae et Mercurii* IX, 979, ed. J. Willis, Leipzig, Teubner, 1983, p. 377.

[8] Cf., e.g., Peter Vossen, 'Über die Elementen-Syzygien', in Bischoff, B. – Brechter, S. (eds.), *Liber floridus: Mittellateinischen Studien*, St. Ottilien, Eos Verlag, 1950, p. 35.

[9] For more details on the reception and development of the Platonic and Aristotelian theories of the properties of elements in the late antiquity and the Early Middle Ages (especially Nicomachus of Gerasa,

III.1. The Properties of the Elements in the Legacy of Plato's Timaeus

Isidore in *De natura rerum*[10] first focused on the three-part classification of the properties of each element according to the Platonic tradition (*DNR* XI, 1) – fire is thin (*tenuis*), sharp (*acutus*) and mobile (*mobilis*); air is similarly sharp and mobile, but unlike fire it is thick (*crassus*); water, like air, is mobile and thick, but it can also be characterised as obtuse (*obtunsa*); earth is blunt and thick, but differs from other elements in that it is immobile (*immobilis*). This means that fire and water are completely separate from each other (*separare*), and their connection (*iungere*) requires the two mediators (*media*), air and water. Attached to this characterisation of the elements is a sketch, the so-called *figura solida*, which is meant to represent the properties of the elements, and which was very often depicted in contemporary manuscripts as well as later medieval manuscripts, although oftentimes in very different ways. Although it is possible to find several forms of this figure and separate them into different classifications, for the purposes of a basic understanding, it should be noted that it was depicted for example as:

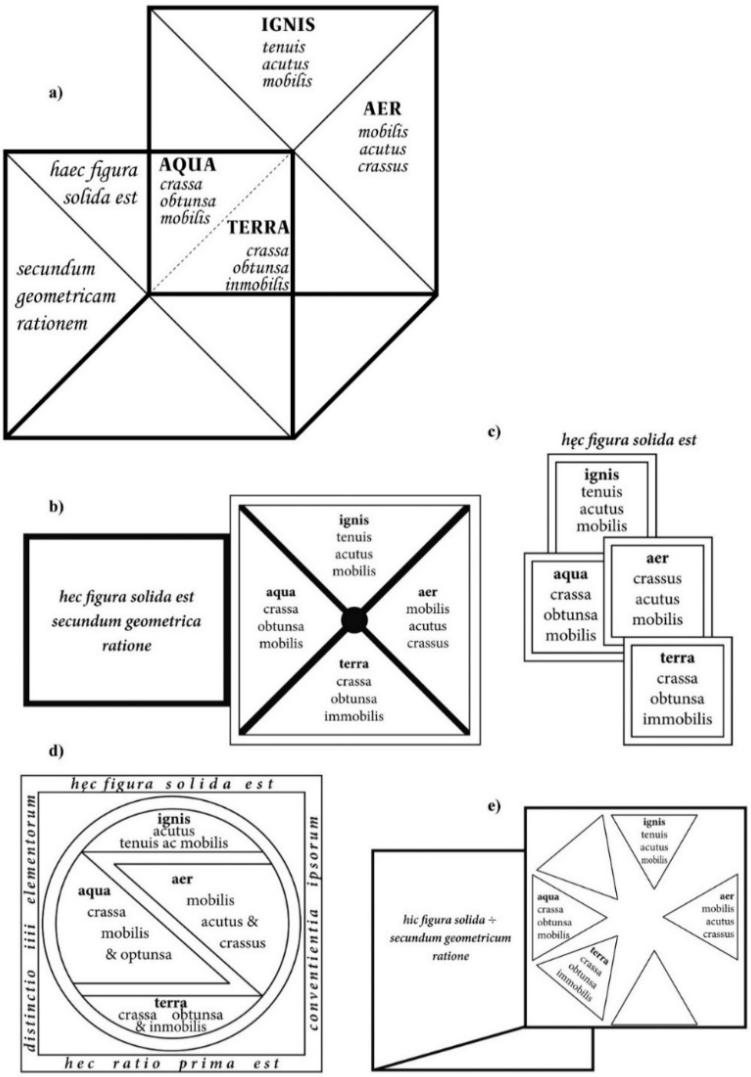
a. some type of a three-dimensional object, such as a cube (see, for example, the manuscripts München, Bayerische Staatsbibliothek, Clm 14300, fol. 7v; London, British Library, MS Harley 3017, fol. 92r; Besançon, Bibliothèque municipale, Ms. 184, fol. 18v; Bern, Burgerbibliothek, Cod. 224, fol. 167v; or Paris, Bibliothèque nationale de France, ms. Lat. 6413, fol. 4v);
b. two quadrilaterals with the elements placed into right-angled triangles (see for example Città del Vaticano, Biblioteca Apostolica Vaticana, Reg. lat. 255, fol. 6v);
c. four overlapping quadrilaterals (cf. for example St. Gallen, Stiftsbibliothek, Cod. Sang. 238, p. 337);
d. a circle (cf. for example Einsiedeln, Stiftsbibliothek, Codex 360(177), fol. 38v; or Oxford, Bodleian Library, MS. Auct. F. 2. 20, fol. 5v);
e. a somewhat confusing combination of quadrilaterals and triangles (München, Bayerische Staatsbibliothek, Clm 396, fol. 12r).

Apuleius of Madaura, Porphyry of Tire, Ambrose of Milan, Nemesius of Emesa, Macrobius, Calcidius, Proclus, Boethius etc.) see, e.g., Caiazzo, 'Filosofia della natura…', pp. 1064–9; eadem, 'La forme et les qualités des éléments: lectures médiévales du Timée', in Celia, F. – Ulacco, A. (eds.), *Il Timeo. Esegesi greche, arabe, latine*. Pisa, Pisa University Press, 2012, pp. 310–18; Obrist, *La cosmologie médiévale*, pp. 263–304, etc.

[10] Isidore de Séville, *Traité de la Nature. Introduction, texte critique, traduction et notes*, ed. J. Fontaine, Bordeaux, Féret et Fils, 1960 (reprint Paris, Études Augustiniennes, 2002), (hereafter cited as *DNR*).

Cf. Fig. 2, which shows only the properties of the elements. The additional information that is sometimes found in these depictions (such as the Sun and the Moon, various animals, symbols representing individual elements, etc.) is not copied here.

FIG. 2 – VARIOUS MANUSCRIPT VERSIONS OF THE SO-CALLED *FIGURA SOLIDA*



Although the interpretation of this schema is not the main subject of this study it may nevertheless be useful to briefly pause here and suggest two possible interpretations as doing so may help provide insight into how the elements and their properties were viewed at the time, including the connection between their geometric and physics-based interpretations.

John E. Murdoch presented a reading of the *figura solida* according to which the six sides of the cube (*cybus elementorum*) commonly depicted in this schema may represent the six Platonic properties of the elements and the elements themselves are represented by the four vertices of the cube, which is where the three properties typical for a given element meet.[11] Although this interpretation was later criticised (see below) and it does not always corresponds with the shape of the cube as preserved in medieval manuscripts, its theoretical basis may still be interesting.

This is especially true if we look at it from the point of view of the interpretation of the elements described in Calcidius' commentary on *Timaeus*.[12] Calcidius (*In Tim.* I, 21–2) describes (taking inspiration from Plato,[13] and Neoplatonist interpretations thereof[14]) fire as subtle (*subtilis*), mobile (*mobilis*) and acute (*acutus*) and earth as corpulent (*corpulentus*), obtuse (*obtunsus*) and immobile (*immobilis*). These properties are in opposition (*contrario*) to each other and, as a result, the continuity and connection (*continuatio*) of the world requires two mediators (air and water) which, according to the arithmetic art (*arithmetica disciplina*), maintain the interconnection of the elements through analogy

[11] Murdoch, *Album of Science*, pp. 280–1; cf. also Obrist, *La cosmologie médiévale*, p. 276.

[12] Calcidius, *Commentarius in Platonis Timaeum*, in *Timaeus a Calcidio translatus commentarioque instructus*, ed. J. H. Waszink, London – Leiden: Brill ²1975, pp. 57–346 (hereafter cited as *In Tim.*).

[13] Plato, *Timaeus* 31b, or 55d–6b, in *Platonis opera*, vol. IV, ed. J. Burnet, Oxford, Oxford University Press, 1903 (hereafter cited as *Tim.*).

[14] For more details see, e.g., Sarah Broadie, *Nature and Divinity in Plato's Timaeus*, Cambridge, Cambridge University Press, 2012, pp. 173–242; Matthias Baltes, *Die Weltentstehung des platonischen Timaios nach den antiken Interpreten*, vol. 1, Leiden, Brill, 1976, especially pp. 151–7; Ian Mueller, 'What's the Matter? Some Neo-Platonist Answers', in Mohr, R. D. –Sattler, B. (eds.), *One Book, the Whole Universe. Plato's Timaeus Today*, Las Vegas – Zurich – Athens, Parmenides Publishing, 2010, pp. 151–63; Barbara Obrist, 'Le diagramme isidorien des saisons, son contenu physique et les représentations figuratives', *Mélanges de l'Ecole française de Rome. Moyen Age*, vol. 108, 1996, pp. 154–7; Stephen C. McCluskey, 'Boethius's Astronomy and Cosmology', in Kaylor, N. H. Jr. – Phillips, P. E. (eds.), *A Companion to Boethius in the Middle Ages*, Leiden – Boston, Brill, 2012, pp. 62–3; Jacques Fontaine, *Isidore de Séville et la culture classique dans l'Espagne wisigothique*, Paris, Études Augustiniennes, 1959, pp. 657–60 etc.

(*analogia*), a connecting geometric ratio.[15]

Calcidius explains these congruent ratios (*mensura congrua*) in more detail in the previous chapters of his commentary. Of particular importance for our interpretation of the *figura solida* may be an example from *In Tim.* I, 17–19, which describes how the creator (*fabricator*) of the world connected fire and earth through two mediators (*medietates*). The example concerns two equal (*aequalia*) parallelepipeds (*parallelepipeda*), ΑΒΓΔΕΖΗΘ and ΚΘΛΜΝΞΟΠ, which have no faces in common and touch in only one vertex (Θ). Calcidius shows that for them to be connected, two other parallelepipeds, ΔΓΡΣΘΗΤΛ and ΘΗΤΛΞΥΦΟ, must be added.[16]

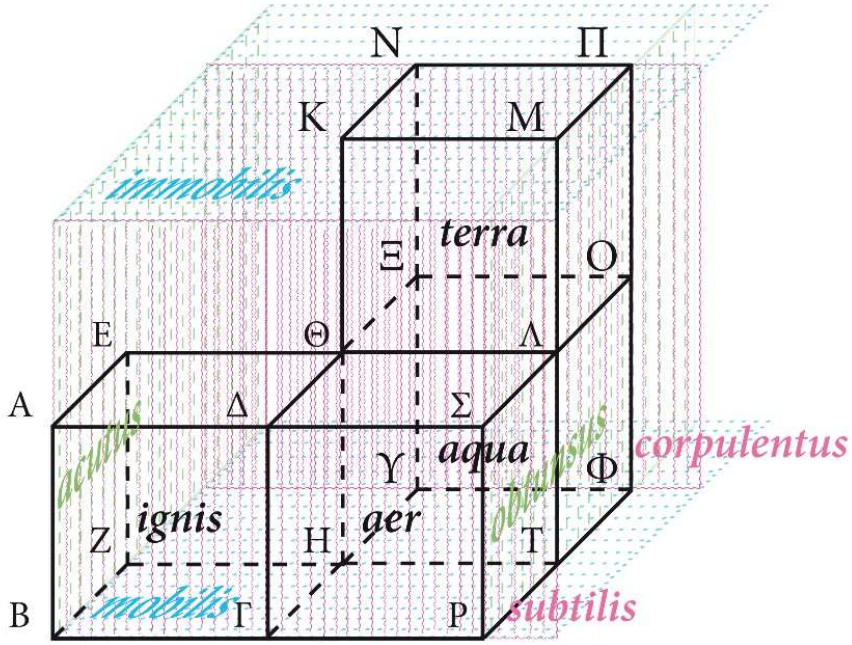
This connection was then described by Calcidius (*In Tim.* I, 20) as a bond that connects different three-dimensional elements – fire as a regular tetrahedron (*pyramis*) and earth as a cube, a regular hexahedron (*cubus*). Even just from the manuscript illustrations[17] representing this transition from fire to earth, it seems that by marking the six planes that delimit the three-dimensional body (in accordance with Murdoch's interpretation), we are offered a possible interpretation of the *figura solida* – see Fig. 3: one side of fire's trapezoid is defined as acute (ΑΒΖΕ), which no other element has, as air (ΗΡΤΛ), water (ΛΤΦΟ) and earth (ΜΛΟΠ) are placed opposite to acuteness and are therefore obtuse; fire (ΑΒΓΔ) and air (ΔΓΡΣ) have one subtle side, while water (ΞΥΦΟ) and earth (ΝΞΟΠ) have one corpulent side; and only earth has one side (ΝΚΜΠ) that is immobile, while the other three elements (water – ΥΗΤΦ, air – ΗΓΡΤ, and fire – ΖΒΓΗ) are mobile.

[15] See further Anna Somfai, 'Calcidius' Commentary on Plato's *Timaeus* and its Place in the Commentary Tradition: the Concept of Analogia in Text and Diagrams', in Adamson, P. – Baltussen, H. – Stone, W. M. F. (eds.), *Philosophy, Science and Exegesis in Greek, Arabic and Latin Commentaries*, vol. I, London, University of London, 2004, pp. 201–20; cf. also Caiazzo, 'Filosofia della natura…', pp. 1066–7; eadem, 'Urso of Salerno on Prime Matter between Plato and Aristotle', in Jacquart, D. – Weill-Parot, N. (eds.), *Substances minérales et corps animés. De la philosophie de la matière aux pratiques médicales (1100–1500)*, Montreuil, Omniscience, 2012, p. 46; Christina Hoenig, 'Calcidius on Cosmic Harmony', in Pelosi, F. – Petrucci, F. M. (eds.), *Music and Philosophy in the Roman Empire*, Cambridge, Cambridge University Press, 2020, pp. 267–73.

[16] Cf. Somfai, 'Calcidius' Commentary on Plato's *Timaeus…*', pp. 212–13.

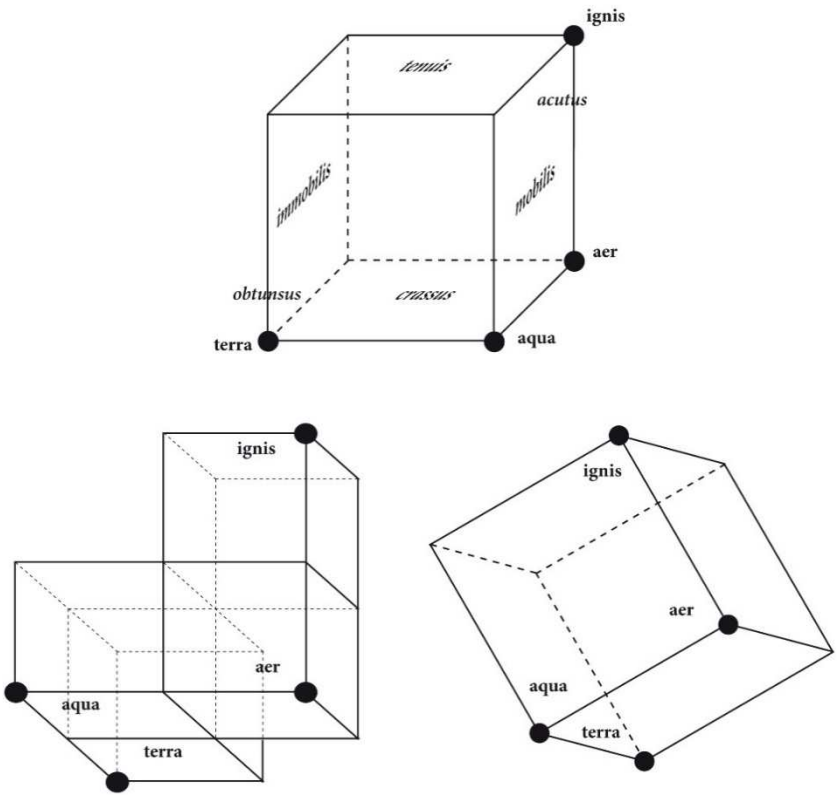
[17] See, e.g., Valenciennes, Bibliothèque municipale, Ms. 293, fol. 22r.

FIG. 3 – ILLUSTRATIONS TO CALCIDIUS' *IN TIM.* I, 18–19



If a cube is formed from individual planes (the six properties of elements) and the individual elements are placed in its four vertices, then a specific rotation of this cube creates a similar schematic structure to the one shown in the various forms of the *figura solida* from Isidore's *DNR* XI, 1 – see Fig. 4. This model opens up an interpretation that connects the Platonic physical properties of the elements with their geometric representation according to Calcidius' congruent ratio.

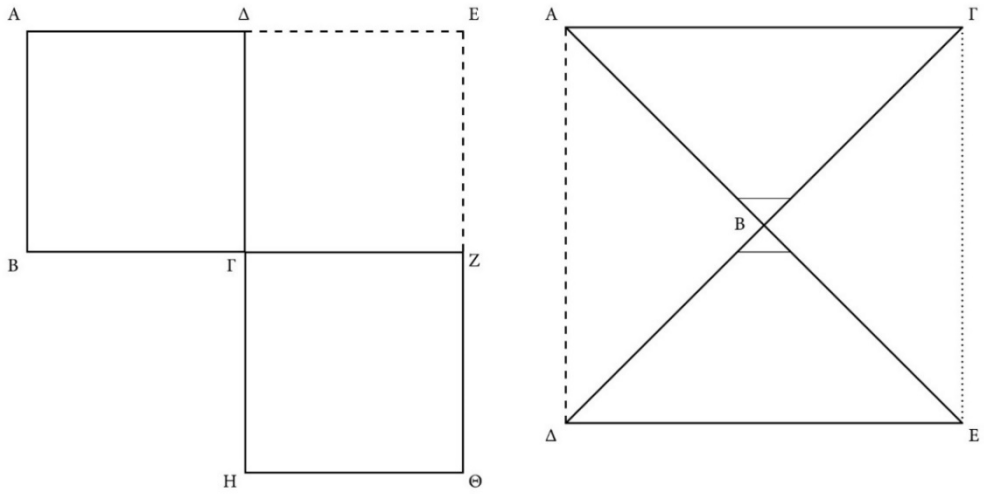
FIG. 4 – CALCIDIUS AS A POSSIBLE INTERPRETATION OF THE *FIGURA SOLIDA*



A similar interpretative key can be used on the form of the *figura solida* that depicts the elements as four triangles forming a square (see, for example, the top portions of Fig. 2a and Fig. 2b above). In his commentary on *Timaeus,* Calcidius also presented the possibility of a harmonic transition between the properties of planar geometric shapes, using the examples of rectangular quadrilaterals and triangles (*In Tim.* I, 11–12) – again, with the use of early medieval manuscripts,[18] these examples can be drawn as depicted in Fig. 5.

[18] Cf., for example, Paris, Bibliothèque nationale de France, ms. Lat. 2164, fol. 28r.

FIG. 5 – ILLUSTRATIONS TO CALCIDIUS' *IN TIM.* I, 11–12



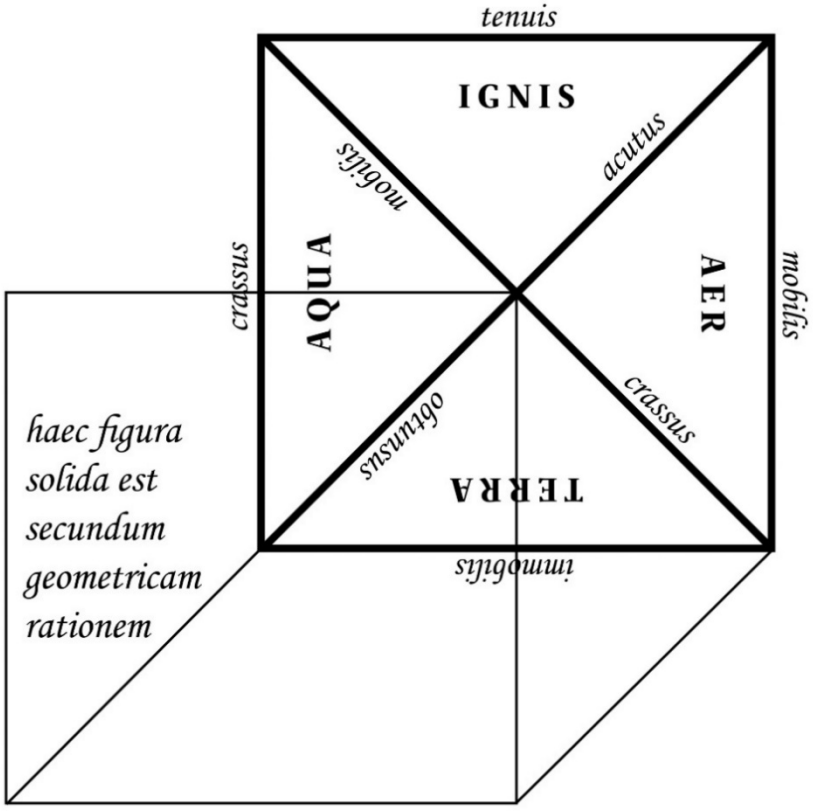
At first glance, it is clear that the form of this *figura solida*, which represents the four elements as isosceles right-angled triangles, corresponds to Calcidius' example, where he looks for a planar mediator between the triangles ΑΒΓ and ΔΒΕ. The manuscript in question illustrates that this mediator is, in accordance with Calcidius' text, the triangle ΔΒΑ, but it also adds the triangle ΕΒΓ, which does, in fact, fulfil the same mediating role.

Bruce S. Eastwood based his interpretation (mainly of the oldest surviving forms of the *figura solida*) on this diagram of the square form of the *figura solida* divided into four isosceles right triangles.[19] At the core of his interpretation is the depiction of the four elements as four isosceles right triangles which can be used to form a square. Each triangle represents one element, and each side of the triangle represents one of the Platonic elemental properties, so that, for each element, all three of its physical characteristics are represented. Since each element always has two properties in common with the elements adjacent to it and one that is different, a natural cycle of these properties is created: fire has

[19] Bruce S. Eastwood, 'The diagram of the four elements in the oldest manuscripts of Isidore's «De natura rerum»', *Studi medievali*, vol. 42, no. 2, 2001, pp. 547–70.

common catheti with air (sharp) and water (mobile), but its hypothenuse is unique (thin); air touches fire (sharp) and earth (thick) and, in addition, it is mobile; earth shares the quality of being thick with air and that of being blunt with water (both catheti), and it is the only element that is immobile (hypotenuse); water then combines with earth (blunt) and fire (mobile) and it has thickness as its hypotenuse. Since this is meant to be a three-dimensional object (*figura solida*), the schema highlights three square areas – the back square with the designation of the elements and their properties; the front square often containing the description *A solid figure according to a geometric ratio*; and finally, the bottom square, which secures the three-dimensional character of the two planar shapes. For an illustration, see Fig. 6.

FIG. 6 – THE *FIGURA SOLIDA* AS FOUR TRIANGLES



This interpretation from Eastwood is clearly supported by the above-mentioned example from Calcidius of a congruent transition between triangles with two potential mediators. Since it can be assumed that Isidore's interpretation of the elements according to the Platonic trinitarian structure of their properties in the form of handwritten diagrams is inspired by Calcidius' commentary on *Timaeus*, which also discusses the same properties, this could lead to the combining of the two-dimensional and three-dimensional depictions of the mediator elements, which would correspond with Eastwood's view of the *figura solida*.

The influence of Calcidius is also evident in the author of *EDQE*, who not only described the same trinitarian Platonic characteristics of the elements as Calcidius, but also – unlike the *figura solida* from Isidore's *DNR* – provides the same list of properties as the one found in Calcidius – cf. see tab. 1.

TAB. 1 – PROPERTIES OF THE ELEMENTS ACCORDING TO CALCIDIUS (*IN TIM.* I, 21), *EDQE* AND ISIDORE (*DNR* XI, 1)

	Calcidius & *EDQE*			Isidore		
fire (*ignis*)	acute (*acutus*)	subtle (*subtilis*)	mobile (*mobilis*)	thin (*tenuis*)	sharp (*acutus*)	mobile (*mobilis*)
air (*aer*)	obtuse (*obtu[n]sus*)	subtle (*subtilis*)	mobile (*mobilis*)	thick (*crassus*)	sharp (*acutus*)	mobile (*mobilis*)
water (*aqua*)	obtuse (*obtu[n]sus*)	corpulent (*corpulenta*)	mobile (*mobilis*)	thick (*crassa*)	blunt (*obtunsa*)	mobile (*mobilis*)
earth (*terra*)	obtuse (*obtu[n]sus*)	corpulent (*corpulenta*)	immobile (*immobilis*)	thick (*crassa*)	blunt (*obtunsa*)	immobile (*inmobilis*)

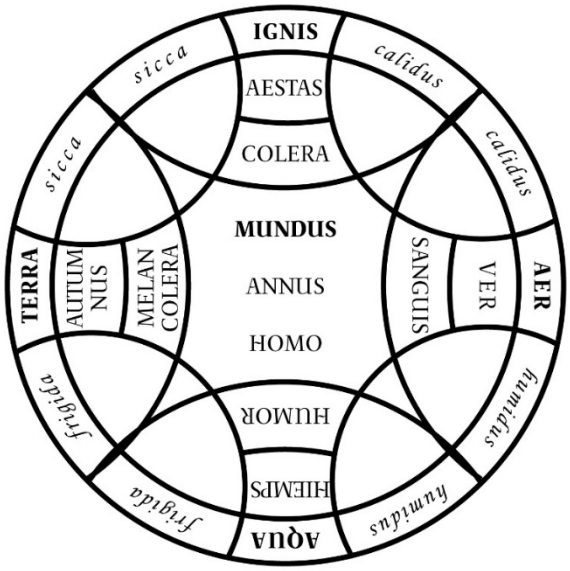
III.2. The Properties of the Elements in the Legacy of Aristotle

In *DNR* XI, 2 Isidore continues by presenting the Aristotelian properties of the elements.[20] For this purpose, he uses an excerpt from the *Hexameron* by Ambrose of Milan, which starts by defining the two basic properties of each element: earth is dry (*arida*) and cold (*frigida*), water is cold (*frigida*) and moist (*humida*), air is warm (*calidus*) and moist (*humidus*), and fire is warm (*calidus*) and dry (*siccus*). Using these properties (*qualitates*), the individual elements are connected (*iugales*) by their mixing (*miscere*) as each subsequent element turns one of these properties into its

[20] See mainly Aristotle, *De generatione et corruptione* II, 2–5, 329b–33a, in *Aristotelis Opera*, vol. I, ed. I. Bekker, Berlin, Reimer, 1831 (reprint coed. O. Gigon, Berlin, De Gruyter, 1960).

opposite.[21] Thus, cold earth and water differ in that the former is dry and the latter moist; water and air are both moist, but water is cold while air is warm; air and fire share warmth, but air is moist, while fire is dry; and similarly, fire and earth have a common property in their dryness, with fire being warm and earth being cold (Isidore, *DNR* XI, 2). By gradually altering these properties, all four elements become transformed and connected by a sort of cycle (*circuitum*). Air is the mediator (*medium*) between water and fire, fire is the mediator between earth and air, forming a harmonic bond. As was noted above, it is this harmony (*concordia*) that gave the elements their name (*στοιχεῖα*), as they are interconnected (*convenire*) and harmonic (*concinere*). The conclusion of Isidore's interpretation (*DNR* XI, 3) is comprised of a circular schema of the elements according to the four Aristotelian properties,[22] the so-called *figura plana*, which also presents its structure as identical with cyclical time (the seasons: the humid and warm spring, the warm and dry summer, the dry and cold autumn, the cold and humid winter) and the four basic humours (blood, yellow bile, black bile, phlegm) – see Fig. 7.

FIG. 7 – *FIGURA PLANA* (*SYZYGIA ELEMENTORUM*)



[21] Ambrose of Milan, *Exameron* III, 4, 18, in *S. Ambrosii Opera*, vol. I, ed. K. Schenkl, *CSEL* 32/1, Praha – Wien – Leipzig, Tempsky – Freytag, 1896, pp. 71–2.

[22] Details can be found in, e.g., Obrist, 'Le diagramme isidorien…', p. 114–26; eadem, 'Wind Diagrams and Medieval Cosmology', *Speculum*, vol. 72, no. 1, 1997, pp. 63–5.

Aristotle's theory of the physical properties of the elements was very well known in the early Middle Ages[23] and it strongly supported the notion of interconnectedness, affinity and harmonic mixing between the elements. While according to the Platonic characterisation of the elements, earth and fire are opposites, according to the Aristotelian view, it is earth and air or fire and water that are opposed to each other and these elements always require one mediator. In any case, the *EDQE* devotes no particular attention to the Aristotelian doctrine – the Aristotelian elemental properties are merely listed in the lower part of the attached schema.

Interestingly, the textual portion of Isidore's *DNR* XI, 2–3 as well as the associated illustrations list the property of dryness differently. In them, earth is characterized as *arida*, while fire is referred to as *siccus*, which is mostly combined into the form of *siccus/sicca* in the pictorial illustrations as a way to express harmonic connection.[24] This difference will be discussed in more detail in section V.2.

IV. MATHEMATICAL CHARACTERISTICS OF THE ELEMENTS

Isidore of Seville (*DNR* XI, 1–3) comprehensively presents the natural properties of the elements, but he makes no mention of any mathematical connections. In contrast, the schema attached to the *EDQE* identifies the individual elements with both geometric shapes and numerical values. Both characterisations can be traced back primarily to the Platonic tradition, especially the already mentioned dialogue *Timaeus*. However, the views of early medieval intellectuals regarding these elemental properties were significantly influenced by the Neopythagorean tradition, as described mainly in Boethius' (480?–524/6) loose translation of Nichomachus of Gerasa's *Introduction to Arithmetic*.

IV.1. Geometric Shapes – Platonic Solids

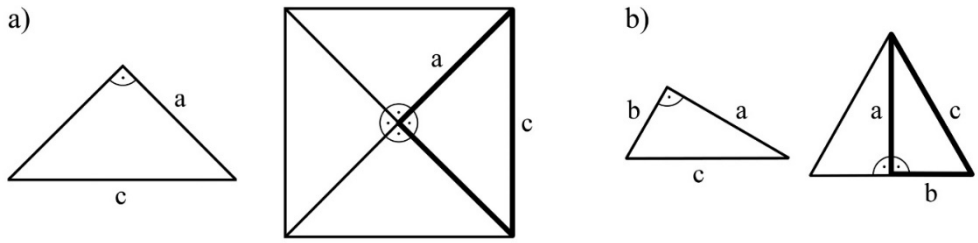
Plato's geometric interpretation of the elements, which is meant to show that the

[23] Cf., e.g., Bede the Venerable, *De natura rerum* 4, in *Bedae Venerabilis Opera*, vol. I: *Opera didascalica*, ed. C. W. Jones, *CCSL* 123A, Turnhout, Brepols, 1975, pp. 195–196 (hereafter cited as *DNR*).

[24] See, for example, the manuscripts München, Bayerische Staatsbibliothek, Clm 16128, fol. 16r; Wien, Österreichische Nationalbibliothek, Cod. 387, fol. 134r; *Città del Vaticano*, *Biblioteca Apostolica Vaticana*, Pal. lat. 1581, fol. 70r; Laon, Bibliothèque municipale, Ms 423, fol. 12r etc.

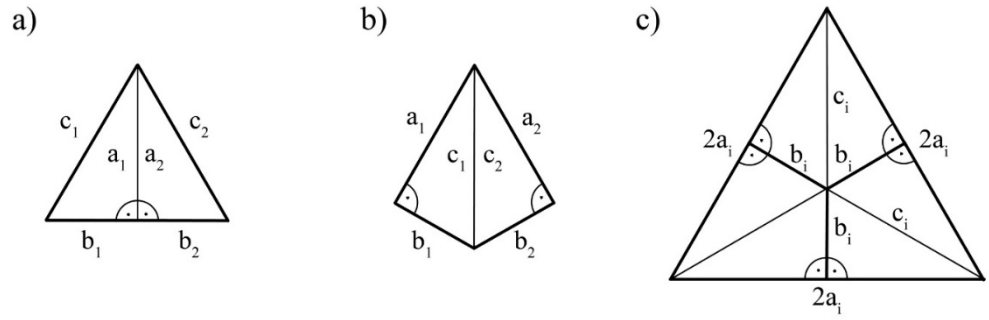
changeable material reality is made out of regular geometric shapes, is based on the notion that since all material bodies have three dimensions, the elements that these bodies are composed of must also be three-dimensional. The surface of solids is made up of flat faces and all flat angular shapes trace their origins to the triangle. In *Timaeus*, Plato derives all triangles from two basic ones: one is an isosceles right-angled triangle, and the other is a right-angled triangle which forms an equilateral triangle if its longer cathetus is put next to that of an identical triangle (*Tim.* 53c–4e). In other words, both basic triangles are right-angled. The first is an isosceles triangle which forms a square when its right angle is put next to those of three identical copies. The second is a scalene right-angled triangle which forms an equilateral triangle if its longer cathetus is put next to that of an identical triangle.

FIG. 8 – THE TWO BASIC TRIANGLES ACCORDING TO PLATO'S *TIMAEUS*



All four elements then arise from both of these right-angled triangles. Three of them have their origin in the scalene right triangle. This is because when the hypotenuses of such triangles are placed next to each other and the resulting quadrilateral shape is tripled, it forms an equilateral triangle made up of three equilateral triangles or six originally scalene right triangles (*Tim.* 54d–e) – for illustration, see Fig. 9.

FIG. 9 – THE EQUILATERAL TRIANGLE AS A COMPOUND OF THREE EQUILATERAL TRIANGLES AND SIX BASIC SCALENE RIGHT TRIANGLES



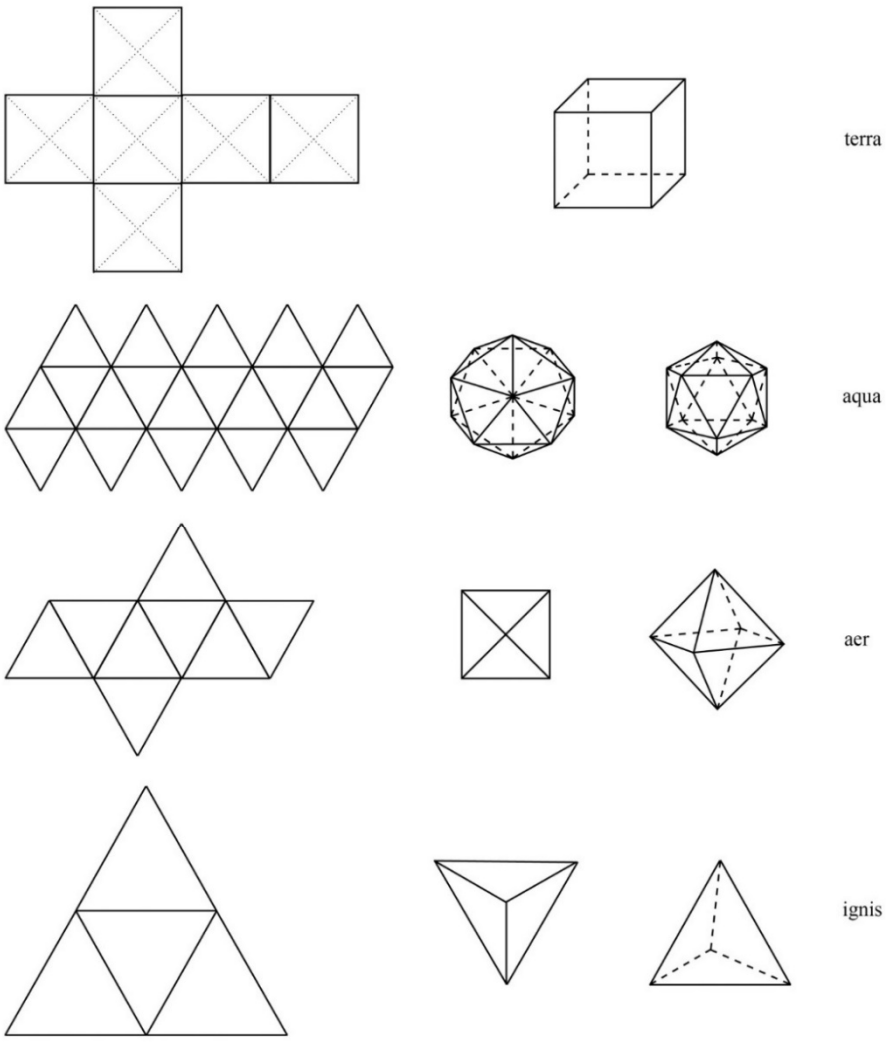
Four identical equilateral triangles form the area of a regular tetrahedron, eight of them make up the shape of a regular octahedron, and twenty make up the shape of a regular icosahedron.

As has already been mentioned, four isosceles right triangles produce a square and six squares (which were originally 24 triangles) form a cube – a regular *hexahedron* (*Tim.* 54e–5c). It is in this way that both types of triangles produce the figures representing the individual elements. The hexahedron (cube), the basis of which is an isosceles right-angled triangle, has a solid and stable surface, so it is stronger and more stable – as such, this is the shape of the element of earth. The scalene right triangle is more mobile, with the icosahedron being the most stable of the three regular polyhedra formed from it – therefore, this is the shape of the element of water. More mobile than that is the octahedron, which is the shape of air, and the most acute and mobile of them is fire, which has the shape of a tetrahedron (*Tim.* 55d–6c).[25] All regular polyhedra (which, together with the

[25] Cf., for example, Alan Code, 'Aristotle on Plato on Weight', in Mohr, R. D. – Sattler, B. (eds.), *One Book, the Whole Universe…*, p. 204; Benno Artmann – Lothar Schäfer, 'On Plato's "Fairest Triangles" (*Timaeus* 54a)', *Historia Mathematica*, vol. 20, 1993, pp. 256–63; Barbara Sattler, 'A Likely Account of Necessity: Plato's Receptable as a Physical and Metaphysical Foundation for Space', *Journal of the History of Philosophy*, vol. 50, 2012, pp. 188–9; Donald Zeyl, 'Visualizing Platonic Space', in Mohr, R. D. –Sattler, B. (eds.), *One Book, the Whole Universe…*, pp. 126–7; Dana R. Miller, *The Third Kind in Plato's Timaeus*, Göttingen, Vandenhoeck & Ruprecht, 2003, pp. 77 or 169–70; Mary L. Gill, 'Matter and Flux in Plato's Timaeus', *Phronesis. A Journal for Ancient Philosophy*, vol. 32, 1987, pp. 50–2 etc.

dodecahedron, form the Platonic perfect solids) can be identified with individual elements – see Fig. 10.

FIG. 10 – THE GEOMETRIC SHAPES OF INDIVIDUAL ELEMENTS (PERFECT SOLIDS)



Calcidius' translation of Plato's *Timaeus* ends just before the geometric section on regular polyhedra. Nevertheless, even in the Early Middle Ages, people were

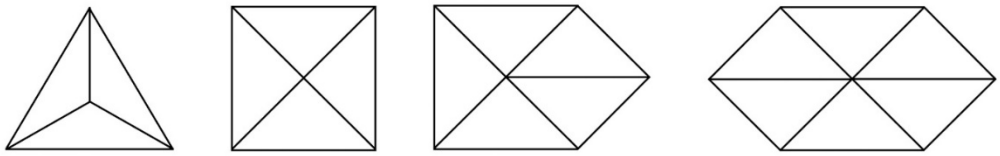
aware of the so-called Platonic solids (see, e.g., *Etym.* III, 11, 2) and their identification with the individual elements. For example, in his commentary on *Timaeus* (*In Tim.* II, 326), Calcidius describes primordial matter (*silva*) and states that the solid (*solida*) and earthy (*terrena*) quality of matter – the element of earth – is represented by a cube (*cubus*); the moist (*humecta*) property – water – is represented by an icosahedron (*icosahedrum*); the gaseous *(spirabilis)* property – air – is represented by an octahedron (*octohedrum*)*;* and the igneous (*ignita*) property – fire – is represented by a tetrahedron (*pyramis*).[26]

However, in addition to the reception of Plato's *Timaeus*, there was another (and undoubtedly widely used) source available which also described the triangular basis of the material three-dimensional reality – Boethius' *Introduction to Arithmetic*.[27] Boethius (*Arith.* II, 21) had noted that the basis (*principium*) of spatial shapes (*solidae figurae*) lies in the pyramid (*pyramis*), which may well have different bases, but the primary is the tetrahedron, which is a pyramid with a triangular base. Almost all three-dimensional geometric shapes can be derived from the tetrahedron, as the tetrahedron is comprised of triangles which form the basis of two-dimensional shapes. Boethius continues (*Arith.* II, 21) by stating that, in geometry, it holds true that any flat polygonal shape (*figura*) has a triangular (*triangularis*) basis, because when you draw a line (*linea*) from the centre (*medietas*) of any basic polygon to the vertex of each angle (*anguli*), these lines will form triangles of which the polygon in question is comprised. If it is a quadrilateral, it will be four triangles, if it is a pentagon, then five, etc. – this also applies to the triangle itself. Boethius' *Introduction to Arithmetic* (*Arith.* II, 6) complements this interpretation with illustrative diagrams – see Fig. 11.

[26] Cf. Jacobus C. M. van Winden, *Calcidius on Matter: His Doctrine and Sources*, Leiden, Brill, 1965, pp. 184–5.

[27] Boethius, *De arithmetica* II, 21, eds. H. Oosthout – J. Schilling, *CCSL* 94A, Turnhout, Brepols, 1999 (hereafter cited as *Arith.*).

FIG. 11 – THE TRIANGULAR BASIS OF POLYGONS ACCORDING TO BOETHIUS' *INTRODUCTION TO ARITHMETIC*



From this follows that matter always has three dimensions and it is three-dimensional shapes which themselves are made up of the elements. Each element has its specific geometric shape, which is also true for the schema from *EDQE*, although, in that text, air is assigned the shape of a sphere (*spera*), instead of an octahedron,[28] which may be related to the documented Carolingian identification of heaven and the celestial sphere with the element of air.[29]

IV.2. Arithmetic Properties of the Elements – Numbers and Numerical Ratios

For the completeness of Plato's interpretation of the metaphysical-mathematical foundations of material reality according to the *Timaeus* dialogue, it is necessary to add that Plato distinguishes between three basic constitutive points of reality. On the one hand, there is a created world of individual changeable specific things, which are made up of four basic elements and can be said to be material, visible and tangible. Furthermore, it is necessary (*necessitas*) for there to be a certain space (*Tim.* 47e), in which the original chaotic matter was initially arranged according to its properties with respect to geometric shapes, thus first creating the four basic elements found in all material entities. And finally, there is the uncreated reality of stability, permanence and intellect which provides examples (*exempla*) in the formation of what has been created, which in many places Plato identifies with harmonic numerical ratios and numerical relationships (*Tim.* 28c–9c).[30]

[28] Cf., e.g., Obrist, *La cosmologie médiévale*, p. 288; Caiazzo, 'Filosofia della natura…', p. 1070; Murdoch, *Album of Science*, p. 351.

[29] See, for instance, *Commentarii et glossae e codice Mettense* ad 4, ed. F. Lipp, in *Bedae Venerabilis Opera*, vol. I: *Opera didascalica*, p. 195; cf. Pressouyre, 'Le cosmos platonicien…', p. 572.

[30] For more details, see, e.g., Gill, 'Matter and Flux in Plato's Timaeus', pp. 37–9; Ann Bergren, 'Plato's *Timaeus* and the Aesthetics of 'Animate Form'', in Mohr, R. D. – Sattler, B. (eds.), *One Book, the Whole*

Similarly, Boethius in the *Introduction to Arithmetic* identified numbers with the example (*exemplar*) which the Creator has in his mind while shaping our world (*Arith.* I, 1), which is a concept often found in Augustine and other influential authors (not only) of the Early Middle Ages.[31] In addition, Boethius in *Arithmetic* also thematises (*Arith.* II, 46) the theory of the Platonic vision of the harmonic ratio, by which the elements of the material world were bound and form the necessary precondition for the existence of material reality. This harmonic ratio is made up of numbers that form the sequence of the so-called Plato's lambda diagram from *Timaeus* (*Tim.* 35b–c),[32] which was well known in the Early Middle Ages, that is, before the year 1000,[33] and has been preserved in several contemporary manuscripts[34] – see Fig. 12.

Universe…, pp. 347–9; Andrew Barker, '*Timaeus* on Music and the Liver', in Wright, M. R. (ed.), *Reason and Necessity. Essays on Plato's Timaeus*, London, Duckworth, 2000, pp. 85–6; Christina Hoenig, '*Εἰκὼς λόγος*: Plato in Translation(s)', *Methodos. Savoirs et texts*, vol. 13, 2013, URL: http://journals.openedition.org/methodos/2994; eadem, '*Timaeus Latinus*: Calcidius and the Creation of the Universe', *Rhizomata. A Journal for Ancient Philosophy and Science*, vol. 2, 2014, pp. 80–110; Gretchen J. Reydams-Schils, 'Meta-Discourse: Plato's *Timaeus* accodring to Calcidius', *Phronesis. A Journal for Ancient Philosophy*, vol. 52, 2007, pp. 314–19.

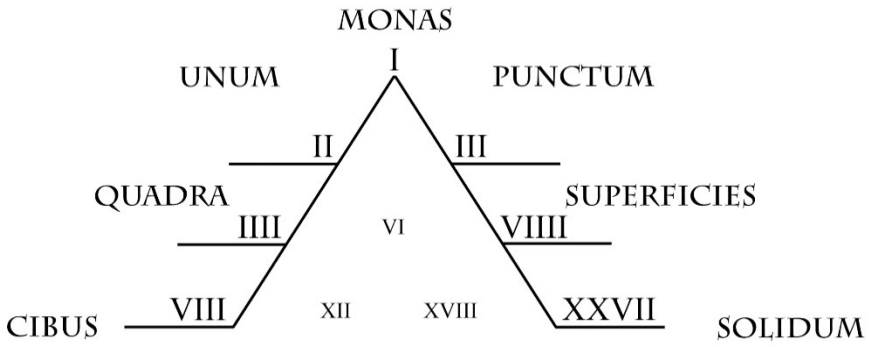
[31] Cf., e.g., Augustine of Hippo, *De libero arbitrio* II, 11, 31–2, ed. W. M. Green, in *Aurelii Augustini Opera*, vol. II, 2, *CCSL* 29, Turnhout, Brepols, 1970, p. 258–9; cf., for example, Junxiao Bai, 'Numbers: Harmonic Ratios and Beauty in Augustinian Musical Cosmology', *Cosmos and History. The Journal of Natural and Social Philosophy*, vol. 13, no. 3, 2017, pp. 192–217; or Luigi Catalani, '«*Omnia Numerorum Videntur Ratione Formata*». A 'Computable World' Theory in Early Medieval Philosophy", in Gadducci, F. – Tavosanis, M. (eds.), *History and Philosophy of Computing*, Berlin – Cham, Springer, 2016, pp. 131–40.

[32] For more details see, e.g., Hoenig, 'Calcidius on Cosmic Harmony', pp. 267–72; Caiazzo, 'Filosofia della natura…', pp. 1076–7; or McCluskey, 'Boethius's Astronomy and Cosmology', pp. 64–5.

[33] Cf., e.g., Macrobius, *Commentarius in Somnium Scipionis* I, 6, 45–6 or II, 2, 15–17, ed. J. Willis, Stuttgart – Leipzig, Teubner, 1994; or in the 10th century Abbo of Fleury and Ramsey, *Commentary on the Calculus of Victorius of Aquitaine* III, 2–3, ed. A. M. Peden, Oxford, Oxford University Press, 2003.

[34] E.g., Madrid, Biblioteca Nacional de España, Vitr. 14-3, f. 26v; Bruxelles, Bibliothèque royale de Belgique, Ms. 9625-26, fol. 17r; Oxford, Bodleian Library, MS. Digby 23a, fol. 25r etc.

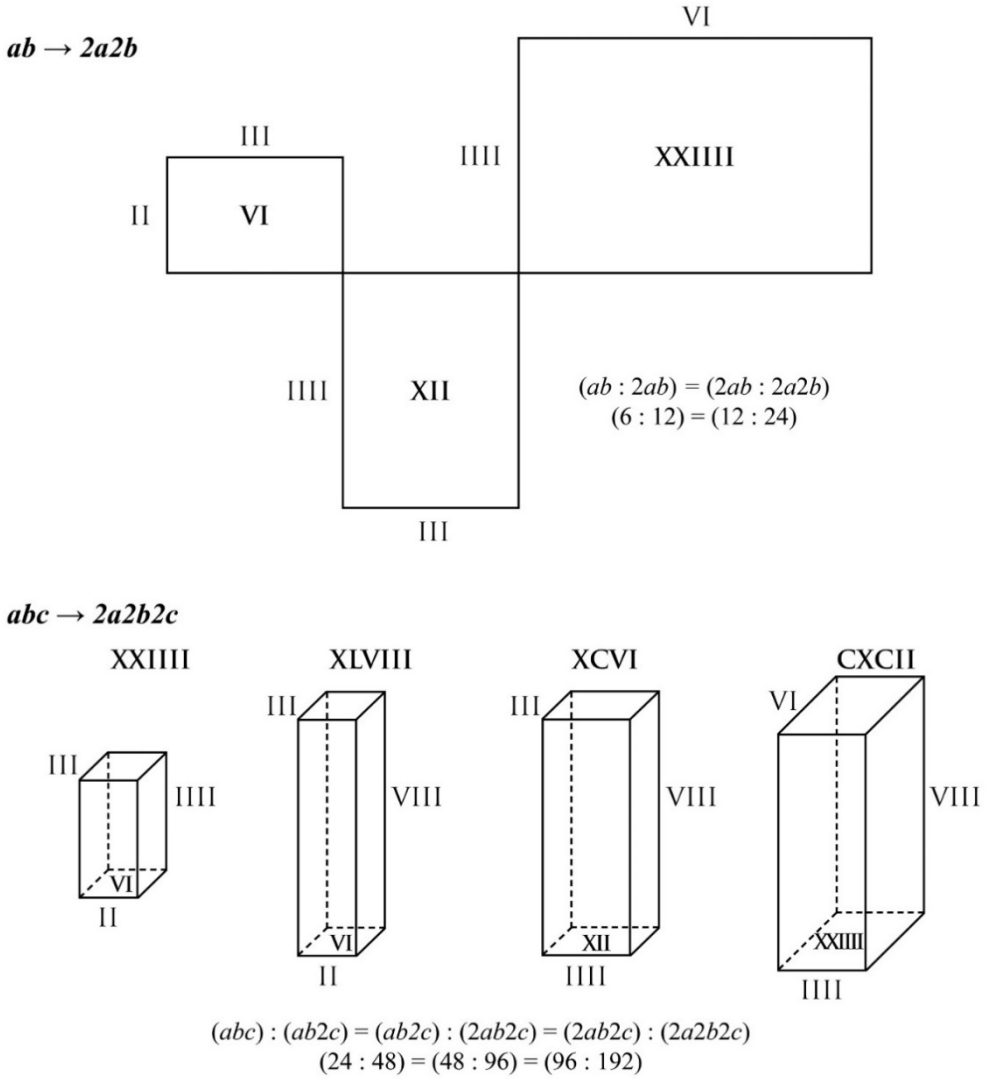
FIG. 12 – THE "LAMBDA DIAGRAM"



Looking at this diagram through the lens of figurate numbers, it is clear that the squares express plane numbers (4, 9), while the cubes express solid numbers (8, 27). In the case of plane numbers, Calcidius (*In Tim.* I, 9) – in keeping with Plato's *Timaeus* (*Tim.* 32a) – demonstrates this on the example of the transition from the rectangular number 6 (sides with lengths 2 and 3) to a double rectangular number (sides with lengths 4 and 6), that is, 24. Such a transition requires one mediator, which is the rectangular number 12 (sides with lengths 4 and 3). But doing the same for solid numbers requires two mediators. Calcidius shows this on cuboids (*In Tim.* I, 14–15), when he goes from the number 24 (length 2, width 3, height 4) to the number 192 (length 4, width 6, height 12), which requires the use of the numbers 48 (length 2, width 3, height 8) and 96 (length 4, width 4, height 8) – see Fig. 13.

This fully corresponds to the words of the anonymous author of the *EDQE*, who mentioned that two mediators are necessary for the transition between fire and earth. These take two lengths of the sides from the nearest element and one from the more distant one (*EDQE* 167, 21–4). This three-dimensional representation then corresponds to the three physical properties of the elements (air shares two properties with fire and one with earth, water shares two properties with earth and only one with fire).

FIG. 13 – DOUBLES OF PLANE AND SOLID NUMBERS ACCORDING TO CALCIDIUS



Although it is rather common to find (early) medieval interpretations that identified the element of fire with the number 27 (3^3) and the element of earth with the number 8 (2^3), for which (according to the schema of the lambda diagram) there are appropriate mediators in the form of the numbers 18 (water)

and 12 (air),[35] in Calcidius' example, it appears that we can assign numbers 24 to fire, 48 to air, 96 to water and 192 to earth.[36]

V. INTERPRETATION OF THE SCHEMA ATTACHED TO THE *EDQE*

Having now a proper contextualisation of the teachings about the elements from the time when the *EDQE* was written, we may turn our focus to the schema of the fragmentary text in question and attempt to interpret it. After looking at the numerical values the schema assigns to the individual elements, I will attempt to interpret these numerical values in such a way as to have them be in keeping with Aristotelian and Platonic physical properties. These findings will then become the basis for a hypothetical interpretation of the numerical values and the outlined connecting lines presented in the upper part of the schema.

V.1. Numeric Values of the Elements

The first relatively surprising aspect[37] of the schema attached to the *EDQE* may be its assigning of the numbers 12 to fire, 24 to air, 48 to water and 96 to earth. While there is a clear similarity to Calcidius (see section IV.2 of this study), the author of *EDQE* halved Calcidius' values.

Justifying why the number 12 is the number of fire is a relatively simple. If we were to look for connections between geometry and arithmetic using Boethius' *Introduction to Arithmetic* (*Arith.* II, 21), it would not be too difficult to come to the conclusion that each polygon is made up of as many triangles as it has vertices

[35] This can be found, for example, in the commentaries on Boethius' *The Consolation of Philosophy* III, m. 9. For editions of the texts and their presentation, see in particular Robert B. C. Huygens, 'Mittelalterliche Kommentare zum O qui perpetua…', *Sacris Erudiri. A Journal of Late Antique and Medieval Christianity*, vol. 6, 1954, pp. 373–427; Edmund T. Silk, 'Pseudo-Johannes Scottus, Adalbold of Utrecht and the Early Commentaries on Boethius', *Mediaeval and Renaissance Studies*, vol. 3, 1954, pp. 1–40; Nicholas M. Häring, 'Four Commentaries on the *De Consolatione Philosophiae* in MS Heiligenkreuz 130', *Mediaeval Studies*, vol. 31, 1969, pp. 287–316; or Hennig Brinkmann, *Mittelalterliche Hermeneutik*, Tübingen, Niemeyer, 1980, pp. 318–47 et seq.; cf., for instance, Irene Caiazzo, 'The Four Elements in the Work of William of Conches', in *Guillaume de Conches: Philosophie et science ai XIIe siècle*, Firenze, Sismel – Edizioni del Galluzzo, 2011, pp. 21–4; Murdoch, *Album of Science*, pp. 354–5; Verboon, *Lines of Thought*, pp. 116–19. For manuscript versions of diagrams with such numerical information, see, e.g., Oxford, Bodleian Library, MS. Digby 83, fol. 3r; Paris, Bibliothèque nationale de France, Lat. 2164, 51v; Paris, Bibliothèque nationale de France, Lat. 15104, 98v or Oxford, St John's College, MS 17, fol. 13r etc.

[36] See Obrist, *La cosmologie médiévale*, p. 268.

[37] Cf. Pressouyre, 'Le cosmos platonicien…', p. 570.

(cf. chap. IV.1. of this paper). The element of fire is represented by a regular tetrahedron, the surface of which is made up of three triangles. And since all three-dimensional shapes that have angles are composed of flat angular shapes – with the basic one of these shapes being a triangle – it naturally follows that a tetrahedron is made up of twelve triangles.

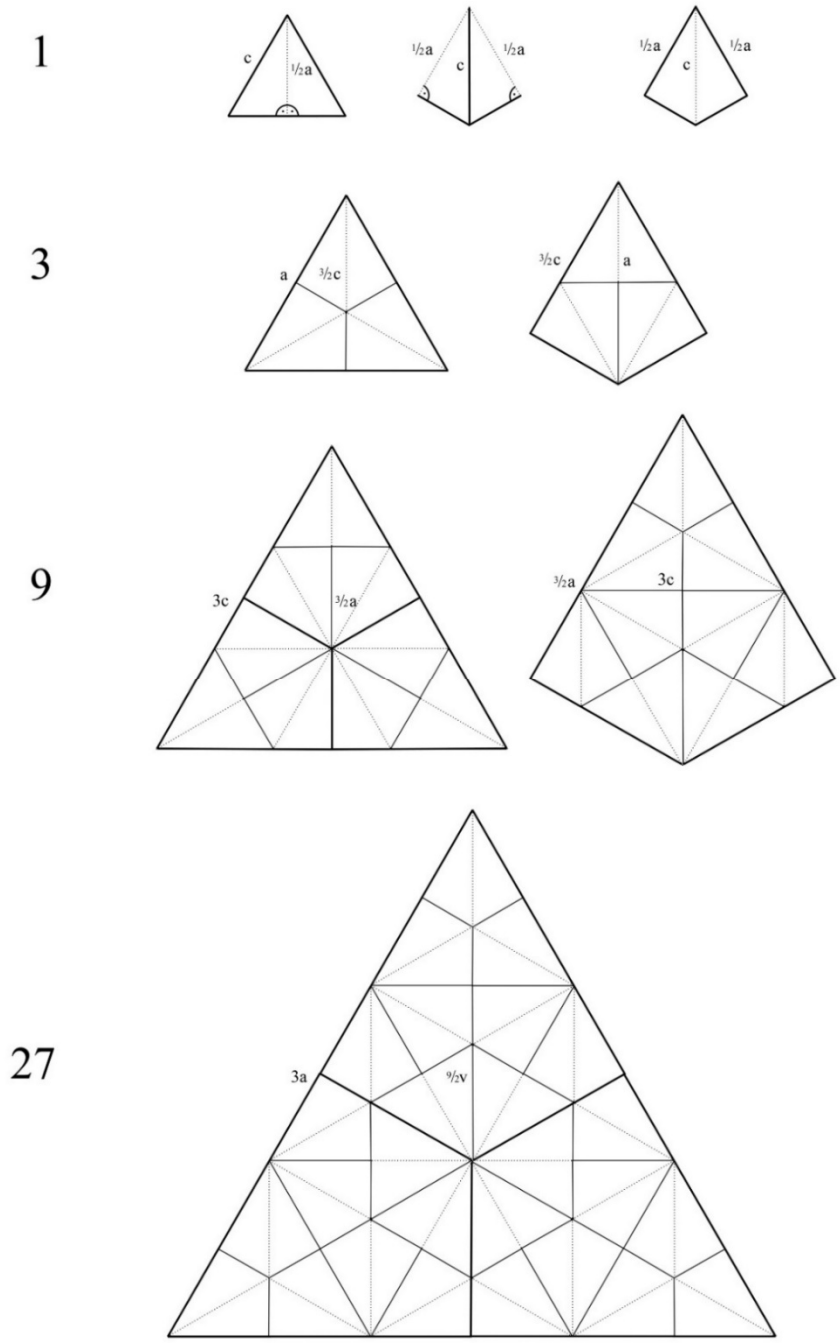
With reference to Plato, it may be noted that these are 12 equilateral triangles, where each of these equilateral triangles arises from two fundamental scalene right-angled triangles (cf. Fig. 8b above). This corresponds to the number 27, which is associated with fire in other (and perhaps surprisingly varied and later) texts.[38] Again, we only need to use Plato's *Timaeus* or, more specifically, the right branch of the so-called lambda diagram: an equilateral triangle with side length *a* can be made from three equilateral triangles with a height of 1/2*a* which can further be divided into six scalene right triangles with the length the cathetus 1/2*a* (see Fig. 9 above). If we want to convert this equilateral triangle with side length *a* into a three-dimensional object, then we must find its cube number, which is how we learn that we need 27 equilateral triangles with a height of 1/2*a* – see Fig. 14.

It then makes logical sense to assign either the number 12 to fire (as in the *EDQE* schema), based on the fact that the area of a tetrahedron is comprised twelve original equilateral triangles, the number 24 (as in Calcidius), which corresponds to the number of scalene right triangles that make up the area of a tetrahedron or, finally, number 27,[39] which corresponds to the number of equilateral triangles we need to three-dimensionally express (make the cube of) the equilateral triangle which makes up the regular tetrahedron. Perhaps influenced by Calcidius, the author of *EDQE* decided to use the numerical value of 12.

[38] Cf., for example, ibid., pp. 561–568.

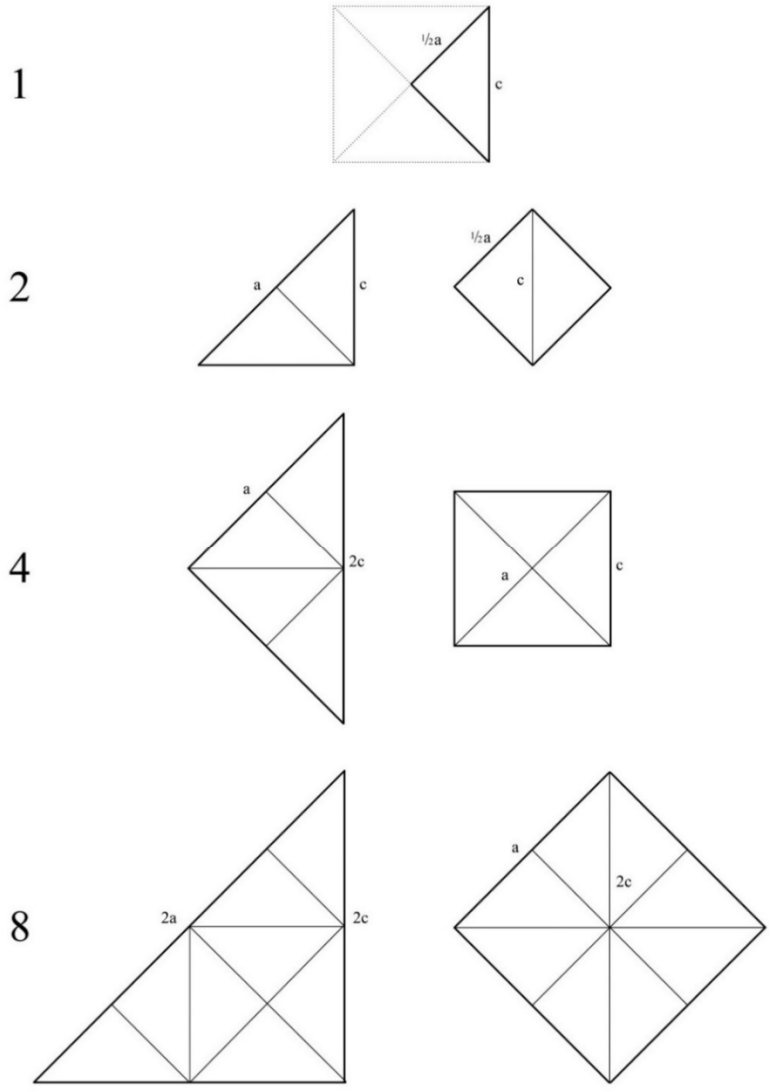
[39] See, e.g., Adalbold of Utrecht, *Commentarius ad Boethii carmen 'O qui perpetua' (Cons. 3, 9)*, in *Serta mediaevalia. Textus varii saeculorum X–XIII*, ed. R. B. C. Huygens, *CCCM* 171, Turnhout, Brepols, 2000, p. 131 (hereafter cited as *In Cons.*).

FIG. 14 – SQUARE AND CUBE NUMBER OF AN EQUILATERAL TRIANGLE



together by their hypotenuses, form a square, but the three-dimensional object requires the cube of the given number, that is $2^3 = 8$, which is therefore a suitable numerical designation of this element – see Fig. 15.

FIG. 15 – SQUARE AND CUBE NUMBER OF AN ISOSCELES RIGHT TRIANGLE



Another number that can naturally be associated with the element of earth would be 24. Earth is represented by a cube, the surface of which is comprised of six squares. Each square (like any other quadrilateral) must include four triangles, as stated in connection to Boethius' *Introduction to Arithmetic*, which also corresponds to Plato's (and Calcidius') characterisation of the basic isosceles right triangle: four of these triangles, if placed together by their right angles, will form a square. Therefore, exactly 24 of these basic triangles are needed to create the six squares that make up the cube.

If the author of the schema in *EDQE* thought in a similar manner, the geometric idea is that the surface of the element of fire is made up of 12 triangles, while that of earth is made up of 24 triangles. Therefore, if we want to show the transition of one element to the other, a 2:1 ratio is a suitable way to express this relationship. If the elements had only two dimensions (making them planar) then the number 12 would be doubled by one mediator: we could go from a rectangle with sides that measure 4 and 3 (total of 12), by doubling the sides through the mediator in the form of the number 24 (rectangle with sides that measure 8 and 3) and arrive at the target value of 48 (rectangle with lengths of sides 8 and 6). However, the elements have three dimensions, so if we are looking for the double of a three-dimensional shape representing the number 12 (for example, a cuboid with side lengths 3, 2 and 2), we need the numbers 24 (a cuboid with side lengths 6, 2 and 2) and 48 (a cuboid with side lengths of 6, 4 and 2), which would give us double of the original object, the number 96 (a cuboid with side lengths of 6, 4 and 4).

By connecting these geometric considerations with the arithmetic application of geometric proportions, it becomes apparent why the schema in *EDQE* uses a 2:1 ratio and why the numerical value 96 is ascribed to the element of earth – this is the value we obtain by doubling the three-dimensional element of fire. It then follows that there must be two mediators between fire (12) and earth (96) – air (24) and water (48).

V.2. Numeric Values of Element Properties

The listed numerical values of all four elements can then be easily converted into their natural physical properties.

If we start from the Aristotelian theory of each element having two typical

properties, then it can be easily assumed that these properties of the elements maintain a 2:1 ratio between each other. Fire (12) is thus the product of dryness (*siccitas*), which is number 3, and warmth (*caliditas*), which is number 4. The elements of fire and air share the property of being warm (number 4), but, in the case of air, dryness changes into moistness (*humiditas*), which can be mathematically expressed as the double of the opposite property – number 6 (3 · 2). Therefore, the number associated with air is 24. This is similar for water, which, like air, is moist (6), but is also characterised by coldness (*frigiditas*), the numerical value of which is double that of warmth – number 8 (4 · 2). Water would therefore be number 48 as the product of humidity (6) and coldness (8).

Earth shares the property of coldness with water but differs from it through its dryness (*ariditas*). This property is (in contrast to the usual form of the so-called *syzygia elementorum* in the *figura plana* – see Fig. 7 above) not called *siccitas* (or *siccus/sicca*), but instead *arida*, which can also be found in other contemporary texts on this topic (see, e.g., Bede, *DNR* 4). This shift may be striking (especially in combination with the schematic representation of elemental transitions), but the reason could likely be explained mathematically, by considering using numerical values that are attributed to the individual properties of the elements and subsequently to the elements themselves as products of those properties. If dryness had the same value as it does for fire (3), then the number associated with earth as the product of coldness (8) and dryness (3) would have to be 24, which would not show a harmonic transition, something that is the very root of the name of the elements. However, it should be noted that dryness closes out the circle, so it is possible to see the dryness of fire (*siccitas*) as physically the same but mathematically different from the dryness (*ariditas*) of the earth, which is therefore expressed as quadruple the original value, 3 . 4 = 12. Earth is then dry (it also has the number (3), but in a mathematically different way (quadruple). Thus, it can be explained that the element of earth is the product of dryness (12) and coldness (8), which corresponds to the numerical value of 96.

Another interpretation could come from presenting the elements as the sum of their properties: fire could be 12 (4 + 8), where the number 4 would represent dryness and the number 8 would be warmth; air is then warm (8) and moist (double of warmth, 16), so it is 24 (8 + 16); water is moist (16) and cold (double of mositness, 32), so it is 48 (16 + 32); earth is cold (32) and dry, but with a different

kind of dryness (again, the difference between *siccitas* and *ariditas*), and since the elements are three-dimensional, it could be expressed by the cube of the dryness of the fire ($4^3 = 64$), so the number of earth is 96 (32 + 64).

Clearly, the stated numerical values of the elements can be made to work with the Platonic theory of the properties of the elements (which assumes three constitutive properties for each element) with only slight modifications. This can be achieved using geometric ratios and cuboid doubling, similar to what was described in relation to Calcidius (see chap. IV.2. above). The element of fire (number 12) can be expressed as a cuboid with the lengths of sides 2, 2 and 3, so its properties can be attributed the values of 2 (acute), 2 (subtle) and 3 (mobile), where $2 \cdot 2 \cdot 3 = 12$. The element of earth is in opposition to that of fire and, as such, it must be attributed opposing properties represented by doubled values: obtuse (4), corpulent (4) and immobile (6), which is why earth has the number 96 ($4 \cdot 4 \cdot 6 = 96$). Between them are the elements of air and water. Air is number 24 as it is obtuse (4), subtle (2) and mobile (3), $4 \cdot 2 \cdot 3 = 24$; water is number 48 as it is obtuse (4), corpulent (4) and mobile (3), $4 \cdot 4 \cdot 3 = 48$.

Thus, the schema in *EDQE* can clearly indicate a mathematical connection with physical reality through the numerical values assigned to individual elements by the author – a potential representation of Aristotelian and Platonic properties and their numerical expression in the elements can be seen in Fig. 16.

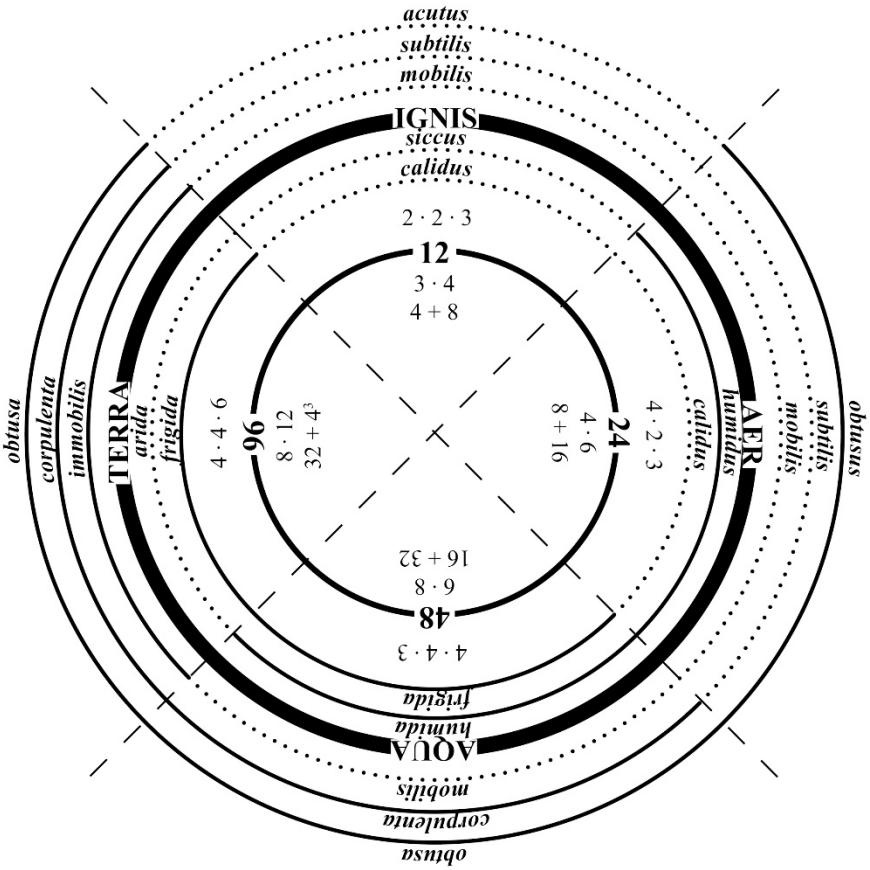
V.3. Numerical Connections between Properties of Elements

The inclusion of three numerical values – 576, 1152 and 2304 – at the top of the diagram, with four lines leading from each of these three numbers (12 lines total), out of which four connect to fire and earth (two each) and eight connect to air and water (four each), may seem complicated or confusing. On an intuitive level, it may be possible that these numerical values express pairs of multiples, with the lines going to the appropriate multiplication products: [40]

$576 = 12 \cdot 48 = 24 \cdot 24;$
$1152 = 12 \cdot 96 = 24 \cdot 48;$
$2304 = 24 \cdot 96 = 48 \cdot 48.$

[40] Cf. Caiazzo, 'Filosofia della natura…', p. 1071.

FIG. 16 – SYZYGIA ELEMENTORUM



An explanation of these multiples and drawn lines could be the brief mention of these oblique lines (*virgulae*) in the text of the *EDQE* itself: they help with comprehending the structure of the three basic properties of the elements and the transitions between them (*EDQE* 168, 14–15), while potentially simultaneously expressing the interconnected (*consertus*) relationships between elemental properties and their other combinations (*EDQE* 168, 15–19).

Looking at the schema through the context suggested by the *EDQE*, there are four elements with three properties each. This means that there is a total of 12 properties, which is equal to the number of the oblique lines. Each line could thus express one of the properties which are represented in the list of

characteristics of the elements in different numbers: the properties of acute (*acutus* – fire) and immobile (*immobilis* – earth) are there only once, the properties of subtle (*subtilis* – fire and air) and corpulent (*corpulentus* – water and earth) are there twice, and three elements are characterised as obtuse (*obtusus* – air, water and earth) and mobile (*mobilis* – fire, air and water).

Similarly, the three numbers (576, 1152 and 2304) in the upper part of the diagram could be references to pairs of the opposing elemental properties – 576 would represent acute and obtuse, 1152 subtle and corpulent, 2304 would be mobile and immobile. At the same time, these numbers would indicate how it is possible to move from one opposite to another in the outlined situation.

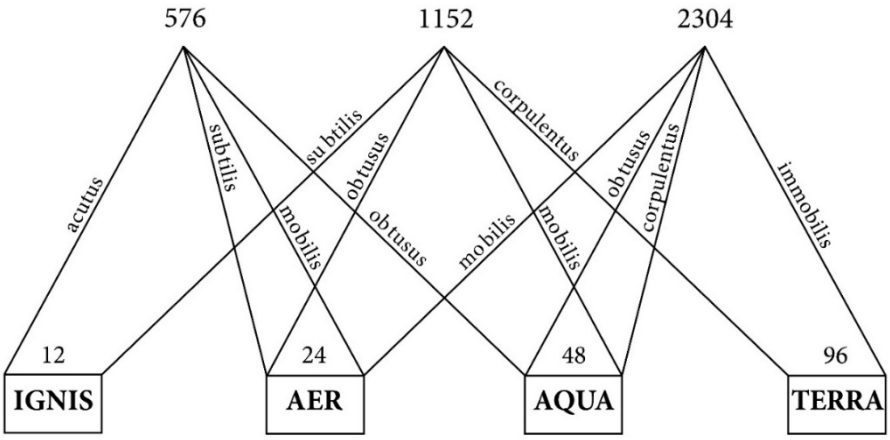
First, let us discuss the number 576. Since it is a multiple of the numbers 12 and 48 and is also the square of 24, one could read the line leading to fire as standing in for number 12 and the property unique to fire (acuteness), while the line leading to water as representing the number 48 and the opposite property of obtuseness. As is clear from what has been stated earlier, if we wish to transition from acute to obtuse, we will need to use two mediators. In this case, these mediators are the properties of subtility and mobility which are specific to air. This could be the reason for why there are two lines going from the number 576 to air, as they express the idea that acuteness and obtuseness are connected through subtility and mobility.

The same is true of the number 1152, from which the lines lead to all four elements. Assuming that, in that case, this is a connection between the opposing properties of subtility and corpulency, then the line leading to fire would have to represent subtility (and thus the number 12) and the line going to earth would represent corpulency (96). The path from subtility to corpulency leads through obtuseness and mobility, which are properties shared by both air and water, so in one case it would mean the number 24 and in the other it would be 48. Thus, the products of the opposites and their mediators would give us the number 1152.

The final number (2304) would, in this reading, also represent the connection of two opposing properties. Since one line leads to earth, these would be immobility (96), which is unique to that element, and the opposite property of mobility (24), represented here by the line leading to air. The mediators of these opposites would thus be the two properties of water (48), namely obtuseness and corpulency.

This interpretation of the schema offers a relatively clear outline of all the elements, their properties and the transitions between them. Each of the upper numbers is a product of the numbers of the individual elements, while always representing the properties that are typical for the given elements – the indicated connections can be more clearly seen in Fig. 17. The end result is better understanding and visualisation of the syzygy of the properties of the elements.

FIG. 17 – PROPERTIES OF ELEMENTS AND TRANSITIONS BETWEEN THEM AS MULTIPLICATIONS



Thus, although the upper part of the diagram may appear confusing and difficult to understand, it can be assumed that the author of both the *EDQE* fragment and the diagram tried to provide a clear and well-thought-out overview. The chosen numerical values are clearly justified and show that it is possible to combine physics-based and mathematical interpretations of the Platonic (as well as Aristotelian) characteristics of the elements.

VI. CONCLUSION

The provided interpretation of the schema found in the *EDQE* makes it possible to clearly interpret all the information presented in it as the essential properties of the elements. The individual elements are likened to geometric shapes – the Platonic perfect solids – which are also used to justify the numerical values assigned to each element. These element numbers can also be viewed as products of their natural physical properties, pointing to the syzygy of elements or the

harmonic bond between numerical ratios. In the Aristotelian view, where each element has two key properties, these are the products (or sums) of the properties, where each following property is represented by a number that is double of the previous one: heat (4) changes to cold (8); and moistness (6) is both double and half of dryness (3 and 12), as dryness is referred to by two different terms (*siccus/arida*), which may be due to an effort to emphasise both the return and the transformation that the properties of the elements underwent. In the Platonic theory, where each element has three basic properties, the numbers would then be arranged according to geometric ratios which also respect the 2:1 ratio, as one side of a three-dimensional geometric shape gets doubled. Therefore, acute, subtle, and mobile fire (2, 2, 3), turns into subtle, mobile, and obtuse air (2, 3, 4), which then becomes mobile, obtuse, and corpulent water (3, 4, 4), which then becomes obtuse, corpulent and immobile earth (4, 4, 6).

It is precisely these properties, with corresponding numerical values in the top part of the schema, which point out not only the need for two mediators between opposing properties (for example, between acuteness and obtuseness, there needs to be subtility and mobility; between subtility and corpulency, the mediators are mobility and obtuseness; and between mobility and immobility, the mediators are obtuseness and corpulency), which can also be interpreted as representing all the twelve properties found across the four elements. The numerical values 576, 1152 and 2304 then convincingly show how the individual things of the material world are composed of these elements, are created by them and, thus, in a certain way, also share their physical properties.

In conclusion, it can be said that the schema attached to the *EDQE* can be read as a comprehensive overview of the physical (Aristotelian and Platonic) and mathematical (geometric shapes and numerical ratios) characteristics of the elements, which aptly captures the basic outlines of how the elements were understood at the time. The fragmentary nature of the accompanying text may have made it harder to comprehend the properties and relationships represented, which, however, does not detract from the importance and interesting nature of the schema itself.

University of Ostrava
Faculty of Arts
Department of Philosophy
Czech Republic
marek.otisk@osu.cz

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