

FUNDAMENTAL MATHEMATICS OF CONSCIOUSNESS

Menas C. Kafatos

ABSTRACT: We explore a mathematical formalism that ties together the observer with the observed in the view that Consciousness is primary, operating through three principles which apply at all levels, the essence of qualia of experience. The formalism is a simplified version of Hilbert space mathematics encountered in quantum mechanics. It does, however, go beyond specific interpretations of quantum mechanics and has strong philosophical foundations in Western philosophy as well as monistic systems of the East. The implications are explored and steps for the full development of this axiomatic mathematical approach to Consciousness are discussed.

KEYWORDS: Consciousness; Quantum mechanics; Monistic philosophy; Western philosophy; Fundamental mathematics; Hilbert space; Qualia; Measurement problem; Subject; Object; Copenhagen Interpretation; Von Neumann; Universal Principles; Complementarity; Recursion; Creative interactivity; Contextuality; Gödel's Theorem; Adjoint; Commutation; Space; Time

INTRODUCTION

The problem of measurement in quantum mechanics (QM) and the role of the observer have been part of quantum theory from the very beginning of its founding but have still not been resolved and remain the central reason for having so many different interpretations of quantum theory, how to take into account measurement and the so-called “collapse of the wave function”. The standard von Neumann (1955) interpretation of orthodox quantum theory, is that the unitary time evolution of the quantum state is interrupted upon measurement and a particular value emerges, given by theoretical quantum probability. What specific value will emerge though, quantum theory cannot predict. Observational choice in the laboratory determines the context of what is to be observed, and we may even presume (as Richard Feynman and John

A. Wheeler would hold) that without observation, quantum systems don't even have any properties. As Wheeler (1981) stated, "no phenomenon is a phenomenon until it is an observed phenomenon". As such, the observer's choices play a fundamental role in the "external" reality that one observes. The observer is an integral part of the process of what is to be observed. Quantum theory opened the door to consciousness but did not provide a solution (Kafatos and Nadeau, 2000; Kafatos, Tanzi, and Chopra, 2011).

Building upon the quantum framework, we realize today that quantum theory has many profound implications for understanding the nature of consciousness. Consciousness continues to challenge all of science even though science has made tremendous strides, e.g. in brain science. Nevertheless, not much progress has been achieved in understanding or even accounting for the most elementary subjective experiences. In fact, many neuroscientists even hold the view that the brain has nothing to do with quantum mechanics, even though they would tacitly agree that quantum mechanics is the foundation of all (physical) reality, *including the brain!* Today, scientists in several polls when they are asked what are the top two most important and unsolved topics facing science, they respond, the nature of the universe, and the nature of conscious experience. Upon reflection, these two profound issues might in fact be closely related to each other.

As such, what used to be in the domain of philosophy and metaphysics, the origin of the mind and in more general terms examining the nature of consciousness and how consciousness arises, can now be approached by science.

However, the issue of consciousness still presents a clear embarrassment to modern science. Despite the great successes of theoretical physics, cosmology and quantum field theory, the advances in molecular biology, brain science, neuroscience and associated phenomena such as memory, anesthesia and quantum brain processes (cf. Pribram, 1966, 1991; Libet et al., 1983; Kafatos and Nadeau, 1991/2000; Hameroff and Penrose, 1995; Roy and Kafatos, 1999; Roy and Kafatos, 2004; Bernroider and Roy, 2005) to just mention a few of the most successful modern scientific fields, we still don't have a comprehensive theory of consciousness that accounts for conscious experience. We even don't seem to agree on a common framework of consciousness-related terms. Yet, any theoretical advance will have to involve an understanding and development of a suitable set of mathematical languages (Kato and Struppa 1999; Kato 2001; Struppa et al. 2002; Kafatos, 2014).

What is lacking is a true dialogue between science and philosophy and the merging of the two. Although science is always based on ontological assumptions (i.e. its foundations are philosophical) most scientists are reluctant to consider the metaphysical assumptions of what they do professionally (Kafatos and Nadeau, 1991/2000). Yet, the role of consciousness has been a central part of the philosophical

discourse not just in the monistic schools of the East, particularly Advaita Vedanta, Shaivism and Buddhism (SwāmīPrabhavānanda and Isherwood, 1975; Chatterji, 1986; Kafatos and Kafatou, 1991; Dyczkowski, 1992, 1994; Pandit, 1997; SwāmīShāntānanda, 2003; SwāmīVimuktānanda, 2005; Singh, 2006; SenSharma, 2007; SwāmīLakṣmaṅjoo, 2012), as well in the great western philosophical systems of Spinoza, Kant, Hegel, and Whitehead and others; and in the ancient philosophies of Heraclitus, Plato, the Neo-Platonists and in the philosophy of the father of philosophy, Socrates himself.

For example, Baruch Spinoza (Morgan, 2002; see also *Wikipedia*):

Spinoza contended that everything that exists in Nature is one Reality, one substance and that there is only one set of rules governing the whole of the reality which surrounds us and of which we are part. Spinoza viewed God and Nature as two names for the same reality, namely a single, fundamental substance that is the basis of the universe and of which all lesser "entities" are actually modes or modifications... Spinoza treats both thought and extension or physical reality as attributes of an ultimate, infinite substance, called "*Deus sive Natura*" ("God or Nature")... His account of the nature of reality, then, seems to treat the physical and mental worlds as intertwined, causally related, and deriving from the same substance. (Ref. *Wikipedia*)

Spinoza's philosophy can be considered as monistic and is in agreement with the fundamental universal principles discussed in the next section.

Immanuel Kant's philosophy views experience as fundamental (*Wikipedia*):

Kant's major work, the *Critique of Pure Reason* (*Kritik der reinen Vernunft*, 1781; see also Kant, 1996), aimed to explain the relationship between reason and human experience... Kant argued that our experiences are structured by necessary features of our minds... The mind shapes and structures experience so that, on an abstract level, all human experience shares certain essential structural features. Among other things, Kant believed that the concepts of *space* and *time* are integral to all human experience, as are our concepts of *cause* and *effect*. One important consequence of this view is that one never has *direct* experience of things, the so-called *noumenal* world, and that what we do experience is the *phenomenal* world as conveyed by our senses. (Ref. *Wikipedia*)

Kant's philosophy tied to experience, supports the idea that qualia, the attributes of experience, play a fundamental role in our views of reality.

Idealism is a central feature of the philosophy of Georg Wilhelm Friedrich Hegel (cf. Redding, 2014):

Hegel's principal achievement was his development of absolute idealism as a means to integrate the notions of mind, nature, subject, object, psychology, the

state, history, art, religion and philosophy. In particular, he developed the notion of the master–slave dialectic and the concept of *Geist* (“mind-spirit”) as the expression of the integration, without elimination or reduction, of otherwise seemingly contradictory or opposing ideas. Examples include relationships between nature and freedom and between immanence and transcendence. (Ref. *Wikipedia*)

Hegel’s philosophy connects to modern complementarity, one of the three universal principles examined in the next section.

The emergence QM had a profound influence in the philosophy of Alfred North Whitehead (1925, 1978). Although originally having developed ideas related to mathematics, in the 1910’s and 1920’s he turned his attention to philosophy of science and metaphysics, departing from most western philosophy. His ideas mesh well with the foundations of quantum mechanics, arguing that reality consists of events rather than matter, that

events cannot be defined apart from their relations to other events, thus rejecting the theory that reality is fundamentally constructed by bits of matter that exist independently of one another. (Ref. *Wikipedia*).

His *Process and Reality* (Whitehead, 1978) forms the foundation of process philosophy. In Whitehead, process philosophy and QM are intimately connected, directly tying philosophy to modern physics.

Having provided some foundations from major Western philosophies, we present below a possible path to integrate these philosophical views with QM and through it with science in general. It is our view that the way forward is to explore mathematical foundations of philosophical statements tied with the subject – object relationships, so central to both the foundations of consciousness-based philosophies; and, through the issue of observational choices, to the foundations of QM itself, from the Copenhagen Interpretation (CI, Kafatos and Nadeau, 1991/2000) to its outgrowth, the orthodox version developed by von Neumann (cf. Stapp 2009; 2012). We also explore how what we have learned from QM may lead to a scientific framework of consciousness. The ultimate goal is to attempt to come up with the *fundamental mathematics of Consciousness*.

The view proposed here is that working with physical theories alone will not lead to a framework of consciousness and such efforts are doomed to fail. The lesson from the quantum view of reality is that the implied world opens the door to mental phenomena through observational choices (cf. Bohr 1934; 1958; von Neumann, 1955; Kafatos and Nadeau, 1991/2000; Stapp, 2009, 2012). Yet, even though consciousness is *implied* in QM, the theory is *agnostic* as to the nature of consciousness. The justification for the approach suggested here is that *any* theory in science is based on mathematics

and, therefore, to get as close as possible to formulate, or at least to attempt to formulate, a scientific view of Consciousness, we must start from mathematics.

THREE UNIVERSAL PRINCIPLES

As we move towards a mathematical formalism of the fundamental relationships between subjects and objects, it is important to understand the common framework that may be applicable to all levels of experience, as revealed primarily by quantum theory but, by far, not limited to it. The world of experiences reveals three fundamental principles which are reflected in quantum theory, in fact all science *and* in the way Consciousness objectifies the world: *Complementarity, recursion and creative interactivity*.

Complementarity (or *Integrated Polarity*) is where ultimately the *apparent opposites become unified at the deeper level* of universal Consciousness. As complementary relations are to be found everywhere, this constitutes an indirect argument that QM is the starting point for developing a scientific framework of consciousness. A consequence of complementarity principle is that it provides horizons of knowledge (Kafatos and Nadeau, 2000; Theise and Kafatos 2013a; 2013b). Boundaries, or horizons of knowledge, *are not absolute*: In von Neumann's picture, they depend on the act of observation.

The second principle is *Recursion* (or *Correspondence*), which can be simply stated, "as here, so elsewhere" (Theise and Kafatos, 2013b). Recursion assures that all particles are similar; all physics laws apply everywhere; all electrons obey the Pauli Exclusion Principle, etc. The world (and therefore Consciousness which is at its foundation) operates through recursive relations.

The third principle, *Creative Interactivity*, provides a framework of interactions at many different levels, universally. Interactions between subjects and objects; between sentient beings (in which case it takes on the special form of *Sentience*); between objects and objects; between cells and cells, etc. etc. In particular, Sentience is in a sense a fundamental aspect of Consciousness, more elementary than conscious awareness experienced by conscious beings.

The three principles *give meaning to the universe*, they are the workings of how Consciousness manifests the universe and apply at all levels, beginning with the fundamental subject – object relationships and the mathematics of Consciousness presented below.

QUALIA

The question of how consciousness arises and if there is an underlying reality based on Consciousness has no answer in any system that takes the division of subject and object as absolute. There are even endless disagreements as to what one means by “consciousness”. We here focus on *qualia* (from the Latin term *qualis*, which means “of what kind”) which are at the heart of an experience-based philosophy of mind. The so-called “hard problem” (Chalmers, 1995) addresses the difficulty of accounting for experience in terms of physical theories and, we content, implies the fundamental role of qualia. Moreover, experience cannot be taken out of a quantum-based ontology, whether the observer is accepted as being essential or whether even denied. The reason is that quantum theory ushered in the role of measurement itself (von Neumann, 1955; Kafatos and Nadeau, 1991/2000; Stapp, 2009; 2012; Kak, 2014).

Erwin Schrödinger (2001) himself held the view that qualia are not material and cannot be accounted by material theories:

The sensation of color cannot be accounted for by the physicist's objective picture of light-waves. Could the physiologist account for it, if he had fuller knowledge than he has of the processes in the retina and the nervous processes set up by them in the optical nerve bundles and in the brain? I do not think so.

Here we advocate a *reasonable* or *common sense* approach: Quantum theory opened the door to consciousness *but* cannot account for consciousness. Simply put, we cannot “take out” the subjective experience from the practice of science. In the end, it boils down as to what are the ontological assumptions (or axioms) of a system of thought. Bohr in the CI argued that QM is silent on this. He opted for epistemology instead. Here we argue that ontology is *implied* in QM and presents with a new vision of reality wherein qualia play a fundamental role (Chopra and Kafatos, 2014):

“Qualia science,” as we envision it, resolves the paradox by showing how the universe operates as the domain of consciousness (Kafatos 2011). An external physical universe as a given is untenable in the post-quantum era; it now requires radical revision as our frame of reference for what is really real and what is not, replaced by the participatory universe that all of us experience through qualia. The process of undercutting the five senses is valid, but we would urge that what makes any experience viable—consciousness—cannot be undercut. This distinction rescues objectivity and subjectivity at the same time, in a complementary relationship.”

OBSERVERS, OBJECTS AND FUNDAMENTAL MATHEMATICS

Quantum mechanics expresses the evolution of a quantum system in terms of the Schrödinger equation which relates the quantum state and its derivative proportionally

to each other (cf. Weinberg, 2013). Quantum field theory (Tomonaga, 1946; Schwinger, 1951) evolved from QM but the main features remained the same.

The Schrödinger equation for the state of the quantum system is written as:

$$i\hbar \frac{d|\varphi\rangle}{dt} = H|\varphi\rangle$$
, where \hbar is Planck's constant, and H is the Hamiltonian which is a measure of the energy of the system.

Although the above formalism is accepted by all practicing physicists dealing with quantum systems as QM and quantum field theory are incredibly successful, the role of measurement, the implied role of the observer, and in fact the reality of the state vector given in the above equation, are not. There are more than twenty interpretations of QM (cf. *Wikipedia*). The von Neumann interpretation assigns a divide between the subject and the object which is exemplified by the “cut” between them. In the standard interpretation, the interaction of the two causes the superposition within the wavefunction to collapse, or in more precise mathematical terms, leads to the “reduction of the state vector”. The standard QM interpretation assigns a fundamental role to observation and as such opens the door to conscious interacting with observed systems. Some other interpretations, such as the transaccional interpretation (Kastner, 2013), and the Orchestrated Objective Reduction (Hameroff and Penrose, 1995) are consistent with the existence of consciousness or assign its origin to quantum processes described by the formalism.

Here we want to go to an even more fundamental level logically *prior* to a physical system being interacted with: We examine the *relationship between a subject and an object*, of which a quantum system being measured is one case. Such relationship can be taken as the prototype of other complementarities and an algebra to deal with general system of complementarities can be developed. In other words, starting from QM, we explore how to go beyond QM to a more fundamental level of existence of subjects and objects.

If we consider the divide of the subject and the object, the question arises how does the divide arise? We cannot define the divide in terms of the strength (or weakness) of the interaction between the two because there are objects or systems that are separated that do not constitute object-subject dichotomous pair. It is true that a collection of systems may be separated in terms of the weak interconnections between them or within a system by the relative autonomy within the subsystems. But that doesn't provide any help in distinguishing a sentient agent from a non-sentient system. The two may or may not be strongly connected.

If one compares the orthodox QM approach to that of decoherence based reduction of the quantum wavefunction (Zurek, 2003), one sees that whereas the

former makes a clear distinction between the observer and the observed, the latter does not.

The principle of creative interactivity provides a framework of universal interactions, which in turn give *meaning to everything* and allows more complex relationships to emerge. These elements give rise to a universally existing sentient reality, which is at the core of Consciousness. Since sentience cannot be located in the body or matter, it should be more fundamental than matter. Indeed, our claim here is that the physical universe emerges out of this fundamental Consciousness. And interactivity leads to *contextuality*, fundamental in QM, as measurements must have a contextual aspect to them. As such, creativity interactivity, along with complementarity and recursion, *gives meaning to the universe*.

Our starting point, our *ontological assumption is axiomatic*: Stated simply is that underlying, *universal Consciousness operates at every level of reality*. It is founded on the fundamental “I-ness”. The basic nature of Consciousness is also basic to each and every one of us: It is the perfect I-consciousness, the I-awareness (Kafatos and Kafatou, 1991). Three principles, on the other hand, allow universal Consciousness, which otherwise would be unmanifest and unknowable, to operate and give rise to all subjective experiences. The Universe is participatory as Consciousness is in partnership, or participation, with everything in it. This participation manifests as sentience *at all levels, in all objects*. The participatory Universe implies that conscious experience is fundamental. It is the experience of universal Consciousness that manifests in countless beings. Finally, at the individual level, experience becomes subjective. Qualia, as we saw above, give rise to all levels of subjective experience and are the fundamental building blocks of the Conscious Universe.

We use the following symbols for the mathematics of fundamental consciousness, utilizing a *simplified Hilbert space convention*, which has the advantage that it brings forward a familiarity with quantum formalism. As quantum mechanics is the only physics we have that fundamentally relates to observation, the connection to Hilbert space is natural. As such,

$\langle \rangle$ is the symbol for absolute undifferentiated Consciousness

$| \rangle$ is the symbol for Subject, i.e. $| \rangle = \text{“I”}$

$\langle |$ is the symbol for Object, i.e. $\langle | = \text{“That”}$

The A symbol is “Am”. We then use the convention $A| \rangle = | \rangle \text{“I Am”}$. While $\langle | A^+ = \langle |$ means “That Am” (note that there is no difference between “Am” and “Is”).

Note that the verb “Am” could also be placed in parentheses, as it is implied in both these expressions, i.e. “I Am”, or “I (Am)”, etc.

“I Am” is existence of Subject; while “That Am” is existence of Object. Subject and Object are *not* yet differentiated, they form the primordial relationship between what eventually will become a separated object from subject, the *fundamental complementarity*. Also, the adjoint A^+ is used in keeping with Hilbert space formalism.

In the above formalism, the only difference between Subject and Object is a sense of direction (or emphasis): In the “I Am”, the direction is from right to left. In the “That Am”, the direction is from left to right. In order to keep that sense of direction, we introduce a right to left arrow in the “I Am”, as $A \left| \right\rangle$; and a left to right arrow in

the “That Am”, as $\left\langle \left| A^+ \right. \right\rangle$. One then obtains statements such as $\left\langle \left| A \right. \right\rangle$

“I (Am) That”, while $\left\langle \left| A^+ \right. \right\rangle$ “That (Am) I”. It can be easily shown that the direction of $\left\langle \left| A \right. \right\rangle$ or $\left\langle \left| A^+ \right. \right\rangle$ doesn’t really matter and that A^+ and A commute, $A^+ A = A A^+ = I$, where I is the identity (or $[A^+, A] = 0$ and $I = \langle I \rangle =$ Absolute, undifferentiated consciousness).

The emphasis of the statement $\left\langle \left| A \right. \right\rangle$ “I (Am) That”, is in the I. We denote this as the logical statement

I (Am) That (I is written in bold for emphasis). In this relationship, as the subjective part of the relationship **I** (Am) That is emphasized, it signifies the *Will* aspect of Consciousness. Before any knowledge (which is to follow) or subsequent to knowledge any action is undertaken, the subject has to be identifying itself with its own will. The next statement is $\left\langle \left| A^+ \right. \right\rangle$ is “That (Am) I” and the emphasis is in That, i.e.

the statement is written as **That** (Am) I. Here, as the objective part of the relationship, **That** is emphasized. Before any action is undertaken, the object has to be identified. Here, it signifies the *Knowledge* aspect of Consciousness. However, in both previous cases, Will and Knowledge, there is no separation, only a (latent) potential of what eventually become a separation between Subject and Object.

The next statement $\langle \overset{\rightarrow}{|} A^+ A \overset{\leftarrow}{|} \rangle$ shows balance: It is the statement of relationship

That (Am) **I** (Am). But as it can be decomposed to $\langle \overset{\rightarrow}{|} A^+ | \rangle \langle | A \overset{\leftarrow}{|} \rangle$, it is recursive,

That (Am) **I-I** (Am) **That** (**I-I** is of course just **I**, i.e. a recursion occurs; in fact **I-I-I...** are all just **I**). The balance between Subject and Object signifies the (potential) for action. But the Subject and Object even in this balanced state, are still One. In action, both the Subject and the Object are balanced, equally weighted.

In the above universal five logical statements

I
That
I (Am) **That**
That (Am) **I**
I (Am) **That**

the Subject and the Object are not separated but are *poised* to move on to separation, which we claim gives rise to manifestation of objects becoming separate from subjects. However, at the first 5 levels, the *experiencing entities are universal*. In fact, they are all One, Subject, Object and the (latent) relationship between them. The five levels described above are found in Shaivism (Singh, 2006; SenSharma, 2007) and as such, the mathematical formalism developed here links with philosophical monism.

All three fundamental principles, complementarity, recursion and creative interaction or sentience, are operating at these 5 levels: Complementarity is operating as the fundamental relationship between Subject and Object. Recursion operates as the relationships can go either way, left to right, or right to left and in fact can repeat forever, always giving rise to $\langle | \rangle$ and in fact $A^+ A$ or $A A^+$ can be repeated arbitrarily many times, resulting always in undifferentiated Consciousness $\langle | \rangle$. And sentience is found in all relationships, the Subject (potentially) senses or interacts with the Object. We emphasize that as no separation has yet occurred, multiple statements like **That** (Am) **I** (Am) **That** (Am) **I**....can in fact repeat forever.

As we move next to the level of breakdown of the above universal relationships, “persistence” of what is pure *Will* gives rise to (limited) will to know and act. “Persistence” of pure *Knowledge* gives rise to (limited ability to) know and then to act. And “persistence” of *Will* and *Knowledge* gives rise to “persistence” of *Action* but in limited form, with limited ability to act. In other words, the same universal statements operate but now in *limited form*. At that point, the Subject and Object become separated and they become many subjects and objects. The subjects interacting with

other subjects and objects now *appear* as differentiated levels of existence, willing (in a limited way) to know (in a limited way) and act (in a limited way). A certain *veiling of Consciousness occurs, which may be manifest in quantum non-locality* (Kafatos and Kak, 2015)

The simple logical statements that we presented above break down, the universal $| \rangle$ and $\langle |$ (equivalent to two vector states in Hilbert space) are now to become limited subjects and limited objects and direction (left to right, right to left) now matters. This is equivalent to symmetry breaking in physics. The mathematics becomes much more complex, instead of $\mathbf{0}$ and \mathbf{I} , we now have an infinite set of vectors, representing infinitely many sentient beings, interacting with each other and sensing objects.

We will explore the mathematics and breakdown in future works. At this point, it suffices to say that what occurs is the logical statement $\mathbf{I} (\text{Am}) \mathbf{Not}$ that, or $\mathbf{That Not} (\text{Am}) \mathbf{I}$. Here the symmetry that applied to the first five levels breaks down. In fact, as we will see, this breaking down may *allow space, time and mass to arise*; as well as limited will (of subjects), limited knowledge (of objects), and limited action (between subjects and objects), *providing an account of qualia*.

At the level of breakdown of the five pure levels, veiled non-locality and cosmic censorship enter the picture (Kafatos and Kak, 2015): Hence, the world appears as classical, composed of separate subjects and objects. However, the general principles of complementarity, recursion, and sentience still hold but now in an infinitely complex set of entities. The universe is conceptually born. *Fundamental mathematics at the first five “pure” levels is the expression of the fundamental principles*. Subsequently, in the manifestation of the universe, Consciousness manifests space-time, and objectified existence, including our own intellect, ego and mind. *These manifestations are all qualia*. The Universe *evolves* out of Consciousness, it is nothing less than Consciousness, in a “condensed” form. Science, through the mathematics of the three principles, is utilized by our minds and intellect to provide the qualia of understanding of our environment.

CONCLUSIONS AND FUTURE WORK

The implications of the present work for the philosophy of mind, science and consciousness theory are many: If Consciousness is primary, then the subject – object relationship has to be formalized mathematically. Qualia play a defining role as they are the agents of experience. The question though arises, would such a mathematical formalism as developed here have limits similar to Gödel’s Theorem? We can speculate how even space-time emerge from the relationships between subjects and objects, when separation breaks the overall structureless, infinite sea of Consciousness. We suggest that the existence of relationship between sentient observers gives rise to a

“measure” of separation, hence *space*. While internal accounting of subjective experiences gives rise to “measure” of change, hence *time*.

Another issue to explore is why the formalism proposed here is more suitable to Consciousness? One possible answer is that the formalism proposed here is the *most* natural as is tied to *direct experience*, which is most basic and most familiar of the human condition.

I acknowledge useful discussions with a number of colleagues, particularly Subhash Kak, Domenico Napoletani and Neil Theise.

Fletcher Jones Endowed Professor of Computational Physics
Chapman University
Orange, CA 92866
mkafatos@gmail.com, menas@kafatos.com

REFERENCES

- Bernroider, G., and Roy, S.(2005), “Quantum entanglement of K ions, multiple channel states and the role of noise in the brain”.*SPIE* 5841-29: 205–14.
- Bohr, N. *Atomic Theory and the Description of Nature*. Cambridge: Cambridge University Press, 1934.
- Bohr, N. *Atomic Physics and Human Knowledge*. New York: Wiley, 1958.
- David J. Chalmers, D.J. (1995), "Facing Up to the Problem of Consciousness". *Journal of Consciousness Studies* 2 (3): 200–219.
- Chatterji, J.C. *Kashmir Śaivism*. N.Y.: SUNY Press, 1986.
- Chopra, D., Kafatos, M. (2014). “From Quanta to Qualia: How a Paradigm Shift Turns Into Science”, *Philosophy Study*, 4, No. 4: 287-301.
- Dyczkowski M.S.G. *The Aphorisms of Śiva*, Albany: SUNY Press, 1992.
- Dyczkowski M.S.G. *Spandakārikā*(“The Stanzas of Vibration”). Varanasi: Dilip Kumar Publishers, 1994.
- Hameroff, S., Penrose, R. (1995), “Orchestrated Reduction of Quantum Coherence in Brain Microtubules: A Model for Consciousness,” in J. King and K. H. Pribram, edit., *Scale in Conscious Experience: Is the Brain Too Important to be Left to Specialists to Study?* Mahwah, N.J.: Lawrence Erlbaum Associates.
- Kafatos, M. (2014). “The Conscious Universe”, in D. Chopra (edit), eBook, Deepak Chopra, Publisher, c/o New York: Trident Media Group LLC.

- Kafatos M., Kafatou T. (1991). *Looking in, Seeing out: Consciousness and Cosmos*. Wheaton, IL: Quest Books
- Kafatos, M., Kak, S.C. (2014). "Veiled Non locality and Cosmic Censorship", *Physics Essays*, 28: 182-187 arXiv:1401.2180.
- Kafatos, M., Nadeau, R. (1990; 2000). *The Conscious Universe*, New York: Springer Verlag, 1990/2000.
- Kafatos, M.(2011) "The Science of Wholeness", in *Analecta Husserliana*, T. Tymieniecka, A. Grandpierre (edit.), Springer Science, Business Media, B.V.
- Kafatos, M., Tanzi, R., Chopra D. (2011). "How Consciousness Becomes the Physical Universe", *The Journal of Cosmology* 14: 3-14.
- Kak, S. (2014). "Observability and computability in physics", *Quantum Matter*, 3: 172-176.
- Kant, Immanuel; Kitcher, Patricia (intro.); Pluhar, W. (trans.). *Critique of Pure Reason*, xxviii. Indianapolis: Hackett, 1996.
- Kastner, R.E. *The Transactional Interpretation of Quantum mechanics*, Cambridge: Cambridge University Press, 2013.
- Kato G., (2001), "Cohomology, Precohomology, Limits and Self-Similarity of Conscious Entity", Vth Conference on structural-phenomenological modeling; categories and functors for modeling reality; Romanian Academy, Bucharest, *Noesis*, XXVI.
- Kato G., Struppa D. (1999), A Sheaf Theoretic Approach to Consciousness, *The Noetic Journal*, 2, No.1.
- B. Libet, et al., (1983), *Brain*, 106: 623 (1983).
- Morgan, M. L. (edit). *Spinoza: Complete Works*. Indianapolis/Cambridge: Hackett Publishing Company, 2002.
- Pandit B.N. *Aspects of Kashmir Śaivism*. Boulder: Utpal Publications, Santarasa Books, 1997.
- Pribram K. *Brain and Perception- Holonomy and structure in Figural Processing*, N.J.: Lawrence Erlbaum Associates Publishers, 1991.
- Pribram, K. H. (1966), "Some dimensions of remembering: Steps toward a neuropsychological model of memory," in J. Gaito, edit., *Macromolecules and Behavior*, New York: Academic Press.
- Redding, P. "Georg Wilhelm Friedrich Hegel," *The Stanford Encyclopedia of Philosophy*, Edward N. Zalta (ed.) <http://plato.stanford.edu/archives/spr2014/entries/hegel/>, Edition Spring 2014.
- Roy S., Kafatos, M., (1999), "Complementarity Principle and Cognition Process", *Physics Essays*, 12: 662-668.
- Roy, S., Kafatos, M. (2004), "Quantum processes and functional geometry: new perspectives in brain dynamics", *FORMA*, 19: 69.
- Schwinger, J.(1951), "Theory of quantized fields I", *Physical Review* 82: 914-27.
- Schrödinger, E. *What is life? : The physical aspects of the living cell*. Cambridge: Cambridge Univ. Press. ISBN 0521427088, 2001.

- SenSharma, D.B. *Paramārthasāra* (“The Essence of Supreme Truth”) of Abhinavagupta (with the commentary of Yogaraja, translation). New Delhi: Muktabodha Indological Research Institute, 2007.
- Singh J. ŚivaSūtras (“The Yoga of Supreme Identity”). Delhi: MotilalBanarsidass, 2006.
- Stapp, H.P. *Mind, Matter and Quantum Mechanics*. Berlin, Heidelberg: Springer Verlag, 2009.
- Stapp, H.P. *Benevolent Universe?* eBook ISBN: 978-1-105-56456-7, 2012.
- Struppa, D. C., Kafatos, M., Roy, S., Kato, G., and Amoroso, R. L. (2002), “Category Theory as the Language of Consciousness,” *Noetic Journal*, 8(3): 271-281.
- SwāmīLaksmānjoo. *Vijñānabhairava*. Culver City: Universal Shaiva Fellowship, 2012.
- SwāmīPrabhavānanda, Isherwood, C. *VivekaChudamani* (“Crest-Jewel of Discrimination”) of Adīśankarā (translation), Hollywood, CA: Vedanta Press, 1975.
- SwāmīShāntānanda. *The Splendor of Recognition*. South Fallsburg: Siddha Yoga, 2003.
- SwāmīVimuktānanda. *Aparokshānubhūti* (“Self-Realization”) of Adīśankarā (translation). Delhi: AdvaitaAshrama, 2005.
- Theise, ND, Kafatos, M. (2013a), “Complementarity in biological systems: a complexity view” *Complexity* **18**, No. 6: 11-202013.
- Theise, N. D., Kafatos, M. (2013b), “Sentience Everywhere”, *Journal of Consciousness: Exploration & Research*, 4, Issue 4: 378-390.
- Tomonaga, S. (1946), “On a relativistically invariant formulation of the quantum theory of wave fields”, *Progress of Theoretical Physics* 1: 27-42
- Von Neumann, J. *Mathematical Foundations of Quantum Mechanics*, translated by Robert T. Beyer, Princeton, NJ: Princeton University Press, 1955.
- Weinberg, S. *Lectures on Quantum Mechanics*. Cambridge: Cambridge University Press, 2013.
- Wheeler, J.A. (1981) in *Some Strangeness in the Proportion*, ed. H. Woolf, Reading, Addison-Wesley Publishing Co.
- Whitehead, A.N. *Process and Reality*. New York: The Free Press, 1978.
- Whitehead, A.N. *Science and the Modern World*. New York: Macmillan Company, 1925.
- Zurek, W.H. (2003). *Reviews of Modern Physics*, 75:
<http://journals.aps.org/rmp/pdf/10.1103/RevModPhys.75.715>.